A theory of cyclopean perception

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A THEORY OF CYCLOPEAN PERCEPTION

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A THEORY OF CYCLOPEAN PERCEPTION

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"Lastly, the permanent rivalry hypothesis has promising advantages for an explanation of the mechanism of depth perception. Verhoeff (1935), Asher (1953), and Kaufman (1963) assume that, although in each point of the visual field the image from one eye is not perceived, it nevertheless contributes to the impression of depth at some other level of functioning. Hochberg (1964) goes one step further, in that according to him there is only one functional binocular field, the piecemeal of the contents of one eye or the other. It is interesting to study the possibility of depth perception as far as it is known, with such a cyclopean field.

But the explanation of the mechanism of depth perception has not been an objective of the present study".

Thus Levelt (1968) ended his thesis in 1965. The section quoted comprises the program of the thesis which is presented here. It is an attempt to unify the basic ideas of Julesz (1971), as incorporated in his model for cyclopean perception, and of Levelt and the authors quoted above. A cyclopean field and its development due to stimulation are formulated mathematically. This serves as the basis for an explanation of several experimental facts in the domain of binocular combination, binocular rivalry and depth perception. It is a psychophysical approach and it will be discussed in Chapter 1 and subsequent Chapters. Before doing this, two related topics are shortly discussed in this introduction. Firstly, the cyclopean system receives its input from the eyes. Thus one cannot bypass discussion of the transformation of the stimulus in the retina (Section 0.1). The second topic concerns the influence of cognitive factors on the cyclopean image (Section 0.2).

0.1. The retinal and the cyclopean system

Light that falls upon a retina is transformed into some neuronal signal and transmitted to the brain. Here the signals from both eyes are composed into one image: the so-called cyclopean image (Julesz, 1971). This composing mechanism in the brain will be called the cyclopean system. The transformations on the retinal stimuli before they enter the cyclopean system, and in so far as they are important for the theory about the cyclopean system, will be described here by two so-called retinal systems: one for each eye (see Figure 0.1.1). Three mutually independent transformations are assumed to take place in the retinal system (Buffart, 1978): *a non-linear one on the stimulus strength*, a spatial transformation, and a temporal transformation. It is doubtful whether one can maintain the independence assumption fully (see for instance Grossberg, 1978). However, for the restricted purpose of the retinal system it can be maintained profitably throughout this thesis. Let r represent a point on a retina and let t represent time. Then a colourless stimulus can be described by a light-intensity function L on r,t (see Section 2.1.1). The non-linear function F - the so-called transducer function - transforms the incoming light - expressed by L(r,t) - into a dimension-free field E(r,t) by

$$E(r,t) = F(L(r,t)) \qquad (0.1.1)$$

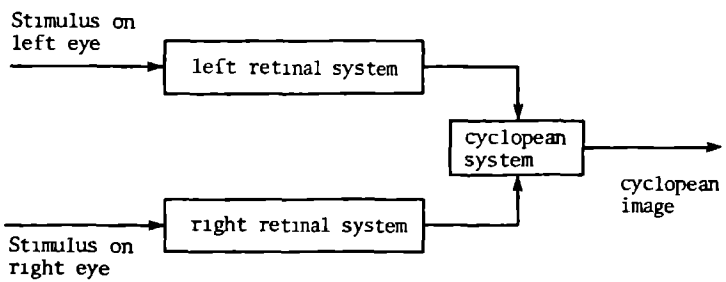

Figure 0.1.1. General schema.

It is assumed (Buffart, 1978) that this transformation is the most important non-linearity in the retinal system. Therefore the following temporal and spatial interactions will be described by linear transformations. As will be argued below, this assumption suffices to describe most observed non-linearities. Thus, if S represents the output field of the retinal system, it can generally be put as

$$S(r,t) = \int_{-\infty}^{\infty}\int\int s(r,t;r',t')E(r',t')dt'dr' \qquad (0.1.2)$$

There is substantial experimental knowledge about s. However, it is not clear if s can be considered as a product of a spatial - s_1 - and a temporal interaction function - s_2. It will be assumed (see also Buffart, 1978) that the independence of the spatial and the temporal interaction exists apart from the principle of maximum-velocity. Then $s(r,t;r',t')$ can be rewritten as

$$s(r,t;r',t') = s_1(r;r')s_2(t;t')Th(t-t' - \frac{|r-r'|}{c}) \qquad (0.1.3)$$

where $Th(x) = 1$ if $x > 0$

and $Th(x) = 0$ if $x < 0$

0.1.1. The non-linear transformation

For F (see (0.1.1)) three types of functions can be postulated. They are all well known from psychophysical measurements of brightness and can be tested theoretically and experimentally. One type, F_1, has been known as Fechner's law

$$F_1(x) = \ln(x/x_1) \qquad (0.1.4)$$

Michels and Helson (1949) introduced a modified formulation of this law by assuming that x_1 is not a constant but depends on some adaptation level. A neurophysiologically more understandable formulation of such an adaptation effect is expressed by the formula of Adams and Cobb (1922):

$$F_2(x) = \frac{x}{x_2 + x} \qquad (0.1.5)$$

The relation between F_1 and F_2 has been plotted in Figure 0.1.2. Apart from the tails both formulae are almost linearly related. If adaptation of x_2 occurs the linear part would be larger. The range of this adaptation parameter is unknown. There are several models of the chemical and electrical processes (Cornsweet, 1970; Grossberg, 1978) which lead to a similar formula F_2. It can explain some characteristics of the perception of hue (Cornsweet, 1970) and also of the saturation-effects in brightness-estimation (see Chapter 4) in the experiments of Jameson and Hurvich (1961). However, the latter type of experiments forced Stevens (1957) to the conclusion that the relation between perceived brightness and stimulus intensity is a power function. Although this relation does not necessarily coincide with the transducer function, many authors (Treisman, 1966, 1970; Treisman & Irwin 1967; Thijssen & Vendrik, 1971; Luce & Green, 1972) make this assumption. In Chapter 4 it will be shown that this assumption (see (0.1.6))

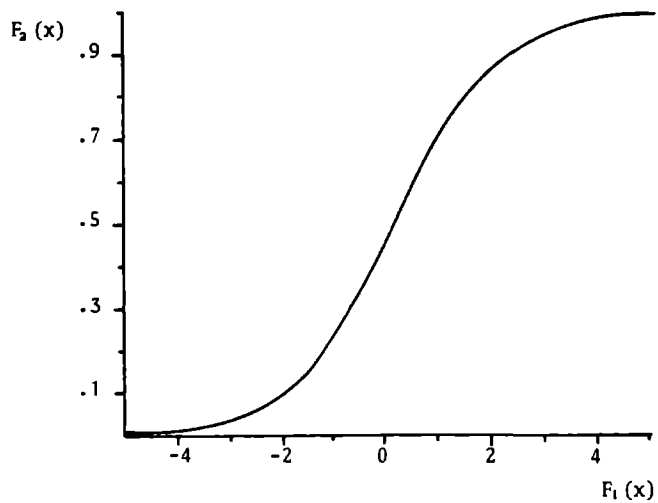

Figure 0.1.2. Relation between $F_2(x)$ and $F_1(x)$.

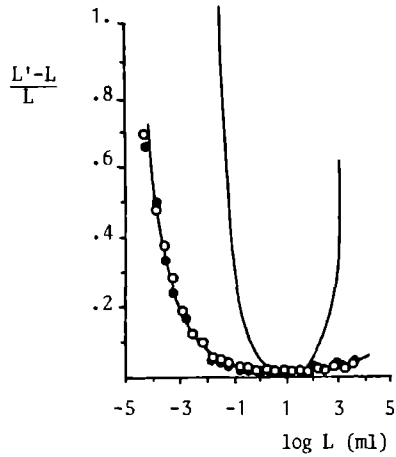

Figure 0.1.3. Theoretical curve derived from $F_2(L')-F_2(L)=1/266$ with $x_2=10$ mL. Data from König and Brodhun (1889).

$$F_3(x) = x^n \qquad (0.1.6)$$

and the second assumption deliver almost identical predictions for binocular brightness matching. However, Stevens' law cannot explain two well known phenomena. Firstly, the saturation effects mentioned earlier cannot be understood. Secondly, Weber's law - $\frac{L'-L}{L}$ is constant -, on which Stevens' law is based, does not hold for all L. It is in fact an U-shaped function of L (König & Brodhun, 1889). The latter can be explained in principle by the quasi-logarithmic transducer function $F_2(x)$ as can be seen in Figure 0.1.3. However, the width of the theoretical curve is much smaller than the width of the experimental one. This difference can be explained by assuming that x_2 in $F_2(x)$ adapts. If this assumption, which has been already introduced above, is true one can calculate the adaptation of x_2 to x from the difference. The adaptation has been plotted in Figure 0.1.4. The curve represents relation (0.1.7a), if L is expressed in mL and L_0 is the adaptation parameter.

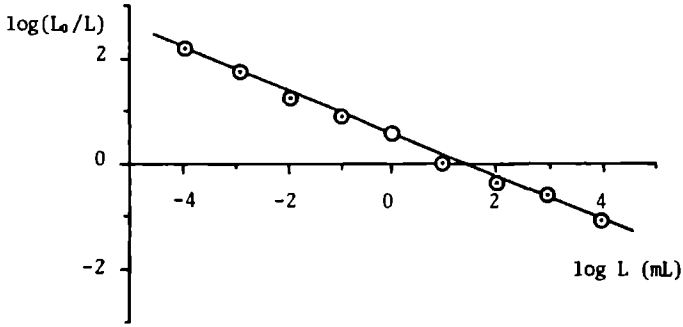

Figure 0.1.4. Adaptation parameter L_0 of F_2=L:(L_0+L) as a function of L. Calculated from experimental and theoretical curves above.

$$\log(L/L_0) = 0.4\log(L) - .53 \qquad (0.1.7a)$$

$$\log(L/L_0) = 0.4\log(L) - .33 \qquad (0.1.7b)$$

If L is expressed in cd/m^2 formula (0.1.7b) holds. Substitution of this adaptation parameter in (0.1.5) delivers

$$F_2(x) = \frac{x}{1x^{.6} + x} \qquad (0.1.8)$$

where 1 is some constant. This expression cannot be interpreted as

$$F_2(x) = \frac{x^{.4}}{1 + x^{.4}}$$

because the adaptation parameter $1x^{.6}$ is supposed to follow the long term changes of the stimulus as the other two terms follow the momentary changes. In a complicated scene the better estimation of the value of x in $1x^{.6}$ is probably the mean overall values of x in space and time.

Analogously to Figure 0.1.2 the relation between $F_1(x)$ and $F_2(x)$ is shown in Figure 0.1.5, but now with adaptation of the parameter x_2 in $F_2(x)$. As a result the linear part of the relationship between both

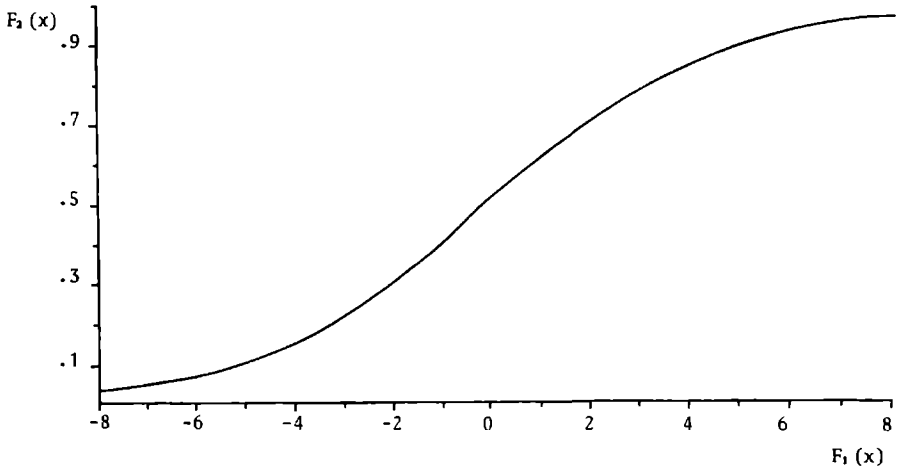

Figure 0.1.5. Relation between $F_2(x)$ and $F_1(x)$ if x_2 adapts.

functions is extended. Thus if the quasi-logarithmic transformation is the true function, one can interpret experimental results as due to a logarithmic transformation. On the other hand, it has been stated above that the transducer functions $F_2(x)$ (with adaptation) and $F_3(x)$ (power function) sometimes deliver identical results in the cyclopean system. This identity is due to an interaction-principle existing in the cyclopean system (see Section 4.4). The comparability of $F_2(x)$ with the two other functions can perhaps explain the difference in results between experiments based on discriminability scales (Fechner's law) and those based on magnitude scales (Stevens' law). Then the first task would concern only the retinal system while the second task would involve higher neuronal centers (see for instance Treisman, 1966).

Naturally, the arguments above are not decisive, apart from the existence of saturation. Other evidence for the quasi-logarithmic function appears from experiments on spatial and temporal interactions. A particular choice for the spatial interaction function - s_1 - and the quasi-logarithmic function together deliver an explanation for some

spatial phenomena (see Section 0.1.2). Therefore it will be used in this thesis as the transducer function.

It is interesting to note that the differences in results in psychophysical scaling also occur for other types of quantities (Engen, 1971). Fechner's law as well as Stevens' law are found to be task-dependent. Moreover both functions, related to these laws, are difficult to understand as transducer functions from a neuronal point of view. Perhaps in all these cases a quasi-logarithmic transducer function at the sensory level and some mechanism in the brain similar to the rivalry mechanism formulated in Chapter 2 are involved.

0.1.2. The spatial interaction

The retina is not a homogeneous space. However, within any peripheral region of some minutes and within the fovea, one may assume homogeneity. The inhomogeneity is evidenced by a decline of visual acuity outside the fovea. Spatial invariance will be assumed within an almost homogeneous region. The supposition that $s_1(r;r')$ equals zero if $|r-r'|$ exceeds such a homogeneous region is supported by its experimental shape (see Figure 0.1.6). Thus $s_1(r;r')$ can best be rewritten as

$$s_1(r;r') = c_1(r,r-r') \qquad (0.1.9)$$

Generally $c_1(r',r)$ is supposed to be a difference of two (distribution) functions on r, or on one of both dimensions, (Ratliff, 1965). The negative function is regarded to correct for the blurring of stimulus-contrast in the retina. It has been shown (Buffart, 1978) that apart from retinal inhomogeneity the spatial interaction in one dimension $c_{11}(x',x)$ can be described by a Lorentz-distribution and a first order correction on its width (see Figure 0.1.6 and (0.1.10)). The Lorentz-distribution causes a mutual influence of retinal points, which blurs the contrast in the percept. The first order correction sharpens this contrast on the basis of the type of blurring. The spatial inhomogeneity can be described by choosing a as a function of x' (see Note 6

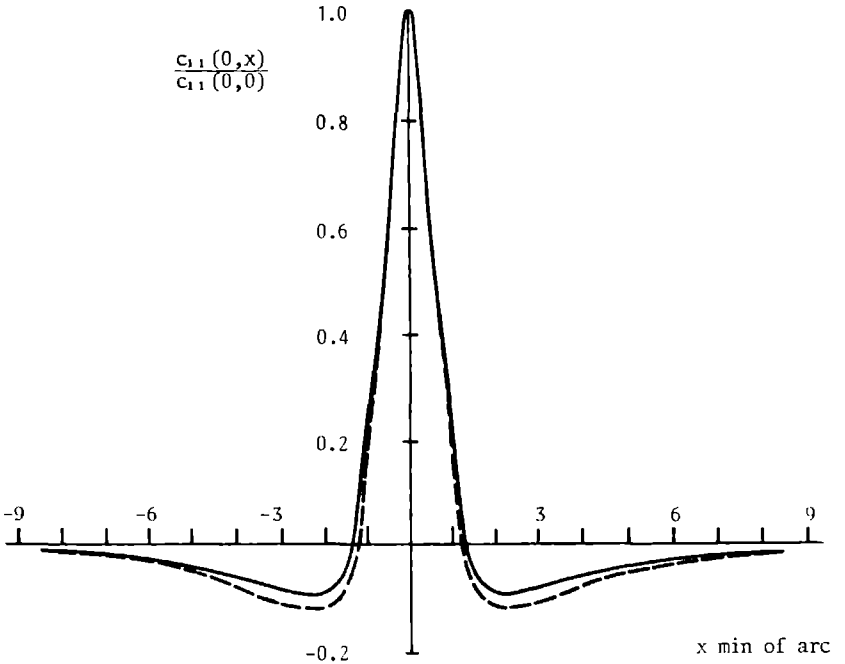

Figure 0.1.6. Lorentz-distribution with first order correction (from Buffart, 1978): a=.02 cycl/deg, (a) b=10 or (b) b=56.

$$c_{11}(x',x) = (1 - ba\frac{\partial}{\partial a})\frac{a}{a^2 + x^2} \qquad (0.1.10)$$

in Buffart, 1978) *). Since the Fourier-transform of (0.2.10) is a linear combination of the two lowest order functions of the orthogonal set of Laguerre-functions (these functions have only positive frequencies), the spatial-frequency domain is described best by this orthogonal set. The Lorentz-distribution does not have a (finite) variance. In

*) The three-dimensional generalization is $\frac{a}{2(a^2+x^2+y^2)^{3/2}}$

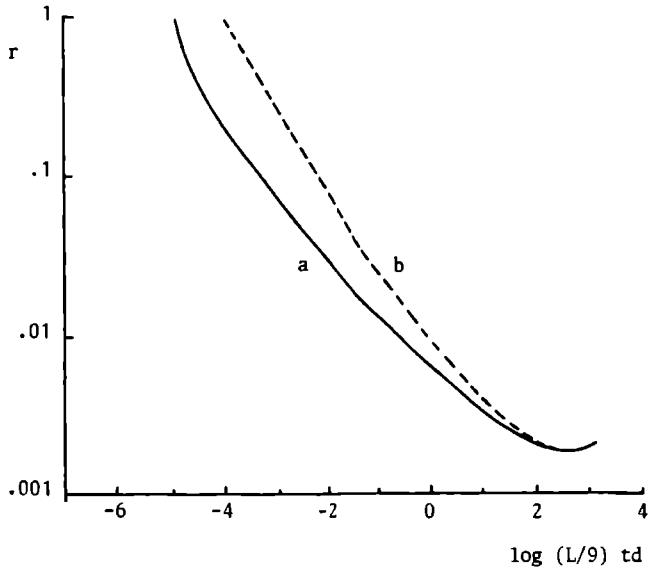

Figure 0.1.7. r as function of L with saturation fixed at L=3333 td and adaptation parameter proportional to $L^{.6}$ (a) or $L^{.5}$ (b).

terms of neurophysiology it implies that, even over a large distance, ganglion cells in the retina show a not negligible interaction.

Many contrast-sensitivity experiments have been carried out with sinusoidal stimuli like

$$L(r,t) = L(1 + r \cos(fx))$$

L is the mean stimulus intensity, r is the contrast parameter. The latter is measured in contrast threshold experiments. The sinusoidal stimulus is not extended over the whole visual field for practical reasons. The width of the field influences the contrast-sensitivity in a special way (Hoekstra, Goot, Brink & Bilsen, 1974). This influence is due to the border contrast of the sinusoidal area (see Section 2.3.1.1). Apart from this, general tendencies of the dependency of r on L and f

can be explained by the quasi-logarithmic transformation. Firstly it has been shown (Buffart, 1978) that if one supposes that a subject looks for a maximum intensity difference in a stimulus, it follows that the measured r will be a series of **)

$e^{af(2m+1)}$ with m is an integer ≥ 0.

Thus if r is a power function of f then it also is a power function of 3f or other odd higher order terms. The third order dependency has been found experimentally (Van Nes, 1968). It demands special attention because such a factor of precisely 3 is probably caused by a (non-linear) device which can be described by a mathematical function before the spatial interaction, rather than being due to two types of spatial interaction. If so, the quasi-logarithmic interaction could cause it. One can prove (analogously to Buffart, 1978) that the logarithmic function (0.1.4) can produce only a power function of f while a proof in the case of the Stevens' like transducer function (0.1.6) - positive or negative - cannot be given. However, the contrast threshold experiments also show a saturation effect (Van Nes, 1968) as a function of the luminance L, which can be explained with the quasi-logarithmic function, but not with the power function. In Figure 0.1.7 the influence of L on the contrast parameter r is shown. It has been calculated from Buffart (1978) with the saturation point at 3300 td. and the adaptation parameter proportional with $L^{.6}$. This exponent has been estimated above (see 0.1.7) from data of König and Brodhun (1889). On the basis of the data of Van Nes (1968) one can argue that the exponent lies somewhere between .5 and .6. However, the spread in the data is too great to be explicit. It seems to depend on the color of the light that is used (Van Nes & Bouman, 1965). The influence of the luminance on the threshold has been

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**) It is essential for the proof that $F_2(x)$ has the form (0.1.8). If x was to be replaced by x^m with $m \neq 1$, the proof would not hold. Naturally, a small departure from 1 is allowed, since the proof then holds in the lowest order of m-1.

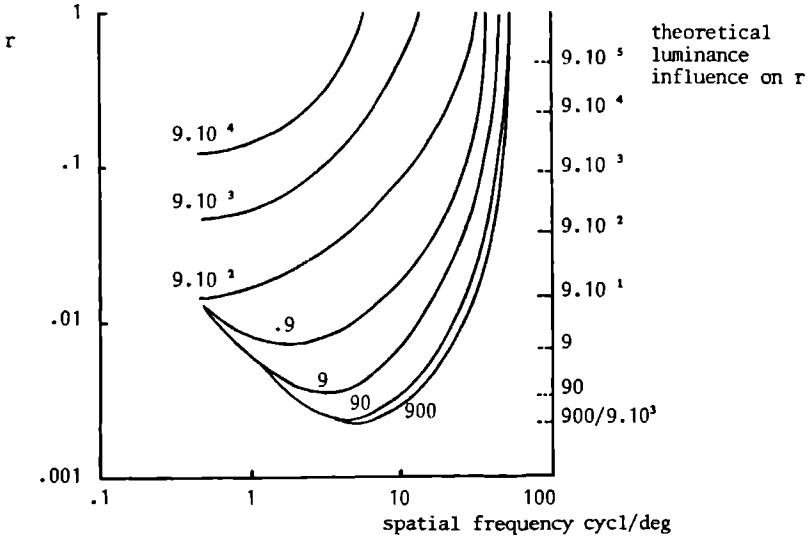


Figure 0.1.8. Relative threshold as function of spatial frequency (data from Van Nes, 1968). At the right-hand side the theoretical influence of the mean luminance level on the threshold is indicated.

calculated also if adaptation is proportional to $L^{.5}$ (see Figure 0.1.7). In Figure 0.1.8 data from Van Nes (1968) are given for green light (525 nm). At the right hand side the influence of the luminance level on the threshold is given for the adaptation proportional to $L^{.6}$. Yet again the experimental results produce arguments in favour of the quasi-logarithmic transformation.

0.1.3. The temporal interaction

As far as the non-linear transformations are concerned many arguments that hold for the spatial interaction also hold for the temporal interaction. In Figure 0.1.9 three experimental results of De Lange

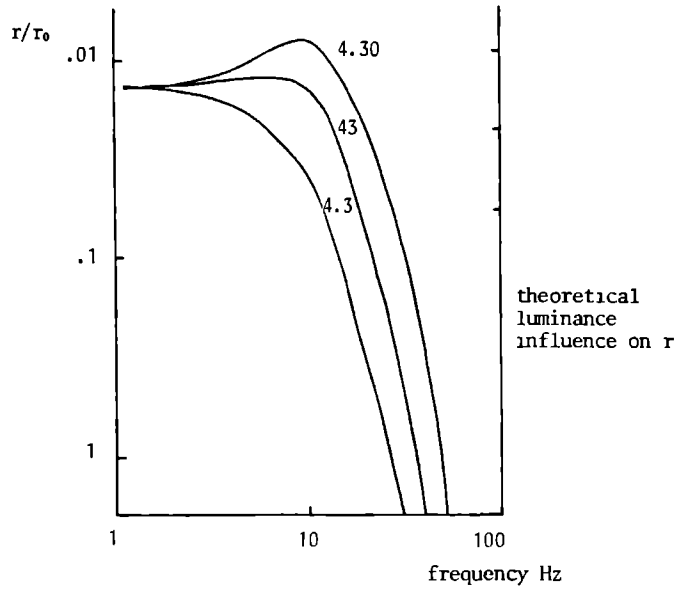


Figure 0.1.9. Relative threshold as function of temporal frequency (data from De Lange, 1957). At the right-hand side the theoretical influence of the mean luminance level on the threshold is indicated analogously to Figure 0.1.8.

(1957) for sinusoidal temporal modulation threshold measurements are given (see also Figure 0.1.8). The theoretically calculated differences due to adaptation to the mean luminance levels are also plotted. Again the differences between the experimental curves are in the order of the theoretical calculations.

It has been argued (Buffart, 1978) that the output of the retinal system depends only on the temporal changes in the retinal input. The dependency on changes can be tested psychophysically by the fading-away of images in the case of stabilized retinal images. It has been found electrophysiologically that only changes in spike concentration of the on-centre and off-centre cells in the retina contribute to contrast and

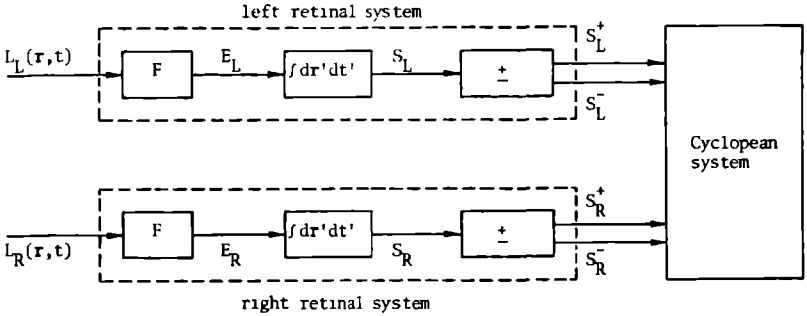


Figure 0.1.10. Detailed schema.

brightness perception (Gerrits & Vendrik, 1970a). If time invariance is assumed, $s_2(t;t')$ (see Section 0.1) can be rewritten as $s_2(t;t') = c_2(t-t')$, where $c_2(t)$ represents some temporal attenuation function (see for instance the curves in Figure 0.1.9) (De Lange, 1957; Kelly, 1964; Ratliff, 1965; Roufs, 1972). The output of the retinal system - S(r,t) - can be positive or negative. When it is positive the quantity

$$S^+(r,t) = \frac{|S(r,t)| + S(r,t)}{2} \geq 0 \qquad (0.1.11a)$$

is positive. When it is negative, the quantity

$$S^-(r,t) = \frac{|S(r,t)| - S(r,t)}{2} \geq 0 \qquad (0.1.11b)$$

is positive. These two quantities are zero otherwise. They are the input of the cyclopean system. (see Figure 0.1.10). They indicate intensity increases and decreases and can be seen perhaps as the psychophysical equivalents of center-on and center-off cells.

The dependency on time-changes causes a non-zero output of the retinal system in the case of stimulus onset or offset and during prolonged stimulation around stimulus contrast if eye-movement occurs. This eye-movement is partly involuntary, partly under subject's control (see Sec-

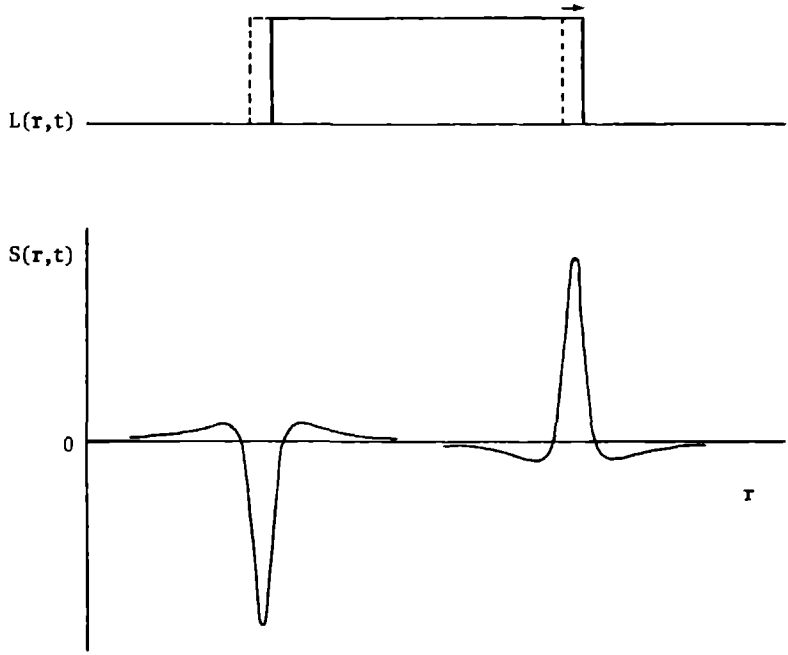


Figure 0.1.11. Type of S(r,t) curve due to unvoluntary eyemovement.

tion 0.2). The involuntary eye-movements are small in comparison with the stimulus field. They cause changes in the neighbourhood of the contrast in the stimulus like the one shown in Figure 0.1.11. A shift of the eye to the left (right) causes at the left contrast a brightness increase (decrease) and at the right contrast the reverse. The signals S^+ and S^- indicate intensity increase or intensity decrease. This difference between increase and decrease is important for the cyclopean system as it will be developed here.

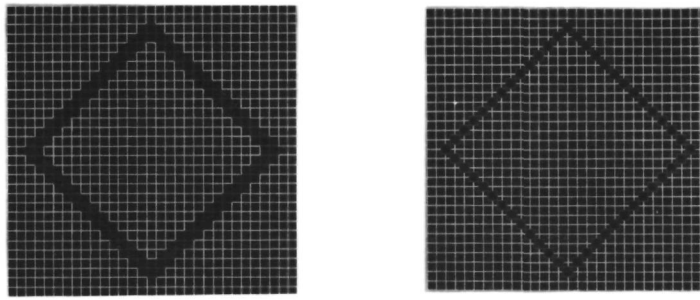
There exists much evidence that one can describe the retinal system as a non-linear transformation followed by a spatio-temporal interaction. In the case of prolonged stimulation this interaction produces an

output S of the type which is shown in Figure 0.1.11. The strength of this S is proportional to the right-hand side of (0.1.12), if the physical contrast is sharp. L_1 and L_2 are the luminances at both sides of the border. If the stimulation is long enough L_0 adapts to some combination of L_1 and L_2; perhaps to $(aL_1 + bL_2)^{.6}$ or to $aL_2^{.6} + bL_2^{.6}$.

$$S \sim \left| \frac{L_1}{L_0+L_1} - \frac{L_2}{L_0+L_2} \right| \qquad (0.1.12)$$

0.2. Cognitive influence

The interpretation of a stimulus influences its brightness impression. This is shown nicely by Van den Brink and Keemink (1976) and can be observed also in Figures 0.2.1a and 0.2.1b. Figure 0.2.1a is an example of the so-called neon-effect (Van Tuijl, 1975). It is the spread of the brightness of one single line over a part of the stimulus. Van Tuijl and Leeuwenberg (1979) have shown that the occurrence of the neon-effect is dependent on the preference for the neon interpretation above other interpretations. In Figure 0.2.1b the neon interpretation is not preferred. Van Tuijl and Leeuwenberg based their assertion on a formal theory of interpretations of visual patterns, Structural Information Theory (Leeuwenberg, 1971). It is pre-eminently a theory about cognitive factors in perception. It predicts preferences between and equivalence (perceptual ambiguity) of interpretations (Leeuwenberg, 1978; Restle, 1979; Buffart, Leeuwenberg & Restle, 1981). The strength of the neon-effect depends on the preference of the neon interpretation in comparison with some other one (Van Tuijl & Leeuwenberg, 1979), although particular luminance relations can also have an influence (Van Tuijl & De Weert, 1979). The retinal system seems to produce the conditions for the interpretations. If these conditions for certain interpretations are absent, they do not occur. The cognitive system chooses an interpretation out of the set of possible interpretations (for which the retinal



a. Neon-illusion. b. No neon-illusion.

Figure 0.2.1. Neon-illusion (from Van Tuijl and De Weert 1979).

conditions are present). Its strength depends on cognitive factors. However, a subject can change his interpretation into another possible one, for instance by making eye-movements or blurring the stimulus. Then the retinal output will change too. Thus the cognitive and the retinal system can work together to strengthen or weaken an interpretation. This schema is depicted in Figure 0.2.2. It explains the combined brightness and interpretation ambiguity in the paper by Van den Brink and Keemink (1976).

Such a loop complicates the interpretation of results of, for instance, threshold experiments or experiments with ambiguous stimuli. So for example in the case of sinusoidal gratings with a finite extension, the border contrast in the cyclopean image probably depresses the effect of the weak contrast within the field by means of the filling-in process (Gerrits & Vendrik, 1970). The depression will be stronger as the density of the weak contrast decreases or the mean luminance level increases. Such a depression can lead to a minor change in eye-movement which changes the output of the retinal system, thereby weakening or strengthening the original effect. Such a model can perhaps make plausible (Buffart, 1978) the occurrence of peculiar effects in

threshold

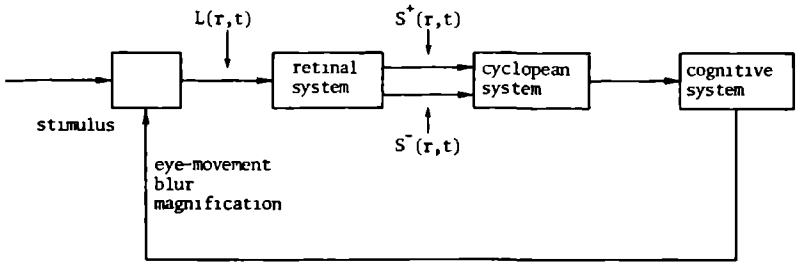


Figure 0.2.2. Influence of the cognitive system on a retinal output. It can be coupled or independent for both eyes.

experiments (Hoekstra et al., 1974).

The existence of such loops may lead to effects which cannot be explained by theories of the retinal and the cyclopean system without making many additional assumptions. Thus the development of a theory which can explain all visual phenomena will only lead to a large ad hoc simulation system. Here the more restricted objective will be approached of developing a theory, which unifies only the known main effects in cyclopean perception on the basis of a set of simple principles.

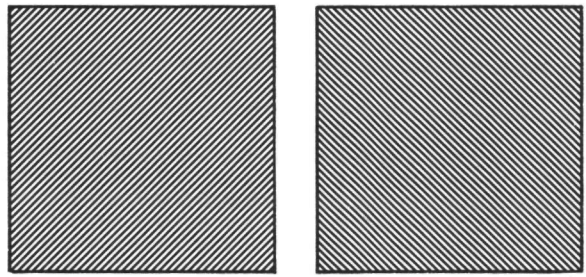


Figure 1.1.1 Rivalry: jigsaw puzzle (after Levelt, 1968).

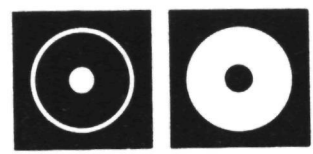


Figure 1.1.2. Rivalry: total suppression (after Levelt,1968).

1.1. The phenomenon of binocular rivalry

Generally, the term binocular rivalry refers to the visual sensations of a subject if both his eyes are stimulated simultaneously by non-identical patterns, and if he cannot combine both stimuli in one stable image. In normal vision one "sees" only single objects, or in other words the two retinal images, half-images, can be combined into one stable image, the "cyclopean" image.

If the patterns in Figure 1.1.1 (Helmholtz, 1866) and those of Figure 1.1.2 (Levelt, 1968) are viewed stereoscopically, both give rise to binocular rivalry. In both cases one uses the identical parts of the half-images to align them properly. So they are used to make one stable image of the identical parts. Except in a single case, as in Figure 1.1.1, in which one can extend the interstimulus difference almost to infinity, one always needs identical parts in the half-images to keep the rivalrous parts of the patterns on corresponding places of the two retinas. One then says that the identical parts are fused. Fusion is an important mechanism in binocular vision. If fusion takes place, the fused parts of the pattern fall upon locally almost corresponding retinal places of both eyes (Panum, 1858; Werner, 1937; Ogle, 1964; Kaufman, 1965). Also the fusion of identical pattern parts is necessary in stereopsis in order to see the other parts of the stimuli in depth (Werner, 1937; Kaufman, 1965; Julesz, 1971; Nelson, 1975). So, every theory about binocular interaction has to incorporate these aspects of fusion.

The rivalry that is experienced by viewing Figure 1.1.1 looks like a jigsaw puzzle whose pieces are chosen at random from one of the two half-fields. These pieces change involuntary in form and origin of

half-field. The rivalry caused by Figure 1.1.2 is an almost complete rivalry between the non-identical parts of the two integral half-images. It is impossible to establish by means of this type of observation, whether there is also rivalry between the identical and fused corresponding parts of the two half-images.

Bouman (1955) introduced an experimental technique for measuring binocular rivalry, which is free from the possible subjectivity in the responses. During binocular rivalry a dim test flash is presented monocularly to a subject. It appears that in a detection task (Fox & Check, 1966; Fox & McIntyre, 1967; Wales & Fox, 1970; Makous & Sanders, 1978, and at variance with it Collyer and Bevan, 1970) the visibility of the flash runs parallel to the visibility of the half-image to which the flash is added. So the subjective reports, in which subjects indicate only which of the two half-images they see, seem reliable. This fact allows us to draw conclusions from experiments that rely more on "subjective" methods, among them the impressive contributions of Levelt (1968) to the experimental knowledge about binocular interaction.

Moreover the "objective" method of Bouman (1955) makes the measurement of rivalry possible even where the "subjective" method fails. So if there is rivalry between two fused identical half-images one can in principle measure it. It has been tried by Fox and Check (1966), Fox and McIntyre (1967) and Makous and Sanders (1978). If such measurements do show that rivalry between fused identical patterns exists, there is rivalry in every pattern. In fact they do show rivalry (see Section 1.2.5). Thus it must be concluded that if there is one mechanism for binocular interaction, rivalry is one of its main characteristics.
In the next Section the role of rivalry in binocular interaction will be examined on the basis of some existing models and their related experiments. From this the view is adopted that permanent rivalry exists and is a central mechanism in cyclopean perception. This view is supported by the fact that the theory to be developed in the next Chapter not only describes the rivalry phenomena, but also predicts the existence of fusion, fusional displacement, diplopia and explains the data on binocular brightness interaction. Furthermore, if the adopted rivalry

principle is generalized to the monocular brightness interaction, predictions can be made for known data in this area. The perception of relative depth can be incorporated in a natural way, without additional assumptions about the interaction mechanism.

1.2. Theories of stereopsis

The theories of stereopsis, that provide explanations for rivalry or depth perception can be divided roughly into three groups: fusion theories, suppression theories and disparity detection theories. Extensive discussions about these theories and experimental results on binocular vision can be found in Levelt (1968), Julesz (1971), Kaufman (1974) and Nelson (1975). One theory from every group will be discussed extensively so that the important contributions of these groups to the understanding of stereopsis as well as the important experimental facts become clear. The theory of Sperling (1970) is discussed as both a fusion theory and as a suppression theory. The model of Julesz (1971) which is described later on, is based on a disparity detection mechanism.

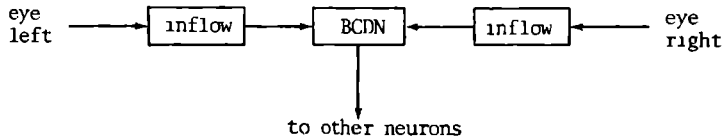
These discussions produce the following conclusions. The theory of Sperling is falsified (Section 1.2.1.3) by an experiment of Levelt (1968). His fusion model, like all fusion theories, cannot solve problems which arise from experiments with random dot stereograms (Section 1.2.2). The falsification of his suppression model, like all suppression models, is based on the fact that the mechanism is one in which the input from one eye suppresses the input from the other eye in some neighbourhood by so-called lateral spread. However, a proposal for a mechanism that strengthens the influence of the input from the same eye within a neighbourhood by lateral spread is not falsified. This proposal is called the permanent rivalry hypothesis. Permanent rivalry is the central mechanism in the theory that is developed in Chapter 2. It states that the rivalry between the input from both eyes has a strictly local character and that the local dominance of one eye is laterally

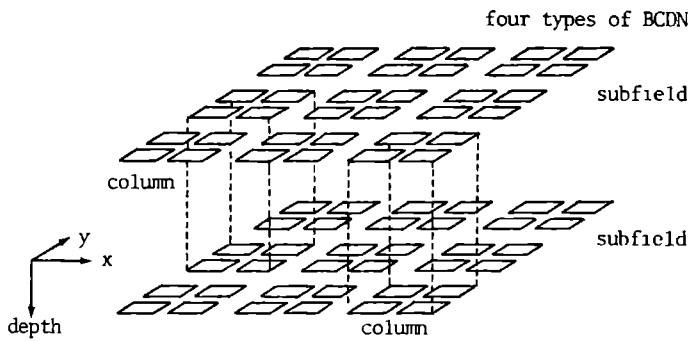
spread (Section 1.2.4). There exists experimental evidence for the permanent rivalry hypothesis. A detailed analysis of experiments of Fox and McIntyre (1967) and Makous and Sanders (1978) shows that the permanent rivalry hypothesis predicts the experimental results where both fusion theories and suppression theories fail (Section 1.2.5). The disparity detection theory of Nelson (Section 1.2.6.4) is falsified by an experiment of Kaufman, Bacon and Barroso (1973). The Julesz model can be formulated (Section 1.2.6.1) such that it is not falsified by this experiment. Moreover, this formulation makes it possible to reformulate the interaction mechanism. In Chapter 2 it will be shown that this reformulated mechanism and the permanent rivalry hypothesis are one and the same mechanism.

1.2.1. The theory of Sperling

This theory is a combination of a fusion model and a model for suppression. As will become clear, they are two, almost independent, mechanisms in the theory, so they can be treated separately. After an introduction to the theory both mechanisms will be discussed. Sperling's theory (1970) consists of a neural and a physical model. Only the neural model is relevant with respect to stereopsis.

Sperling proposes in his neural model a type of neuron that is called a binocular correspondence-detecting neuron (BCDN). He distinguishes four types of BCDNs. They only give an output if excited. They are excited or inhibited by a "receptive field-function". A receptive field-function is some stimulus dependent contour sensitive signal that is carried by the so-called inflow neurons from one of the two eyes into the so-called neural binocular field of which the BCDNs are one of the four types of neurons. The neural system consists of two neural binocular fields. The properties of such a field take care of both fusion and suppression.





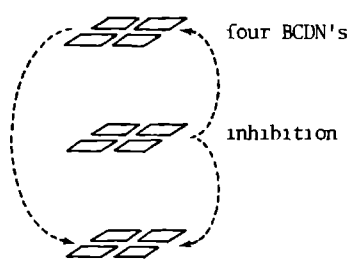


Figure 1.2.1. Fusion mechanism of Sperling's theory (1970).

1.2.1.1. The fusion model

The essential part of the fusion mechanism can be put as follows (see Figure 1.2.1). A neural field is compiled of subfields. These consist of all four types of neurons. These subfields are supposed to be two-dimensional euclidean surfaces parallel to each other in a three-dimensional euclidean space. A stack of BCDNs in this space perpendicular to the surfaces is called a column. Such a column has one BCDN of every type in common with each subfield. The relationship between columns determines the perception of distances in the two-dimensional visual space. The relationship between subfields determines the perception of relative depth. The output of the BCDNs is transmitted to two other types of neurons in the same subfield and inhibits the BCDNs in other subfields but in the same column. By this inhibition one subfield in a column can become dominant. It will be the subfield of the BCDNs with the strongest output. Thus the level of relative depth at a place in the two-dimensional visual space is selected and the sensory fusion takes place.

1.2.1.2. The suppression model

Every receptive field-function has a complementary receptive field-function. Sperling gives no precise definition of a receptive field-function nor of a complementary field-function. Sometimes a receptive field-function is a center-on surround-off field and sometimes a light-dark boundary. The complementary field-functions are the functions in which the light and dark parts are reversed. Another type of receptive field-function is a vertical bar. Its complementary function is, according to Sperling *), a horizontal bar (loc. cit. p. 503). The four

*) The problems that can arise with such a variety of receptive field-functions are shown in Sperling's discussion of the rivalry between a horizontal and a vertical black bar. In his text (loc. cit. p. 509) rivalry is caused by the fact that horizontal and vertical bars

different BCDNs at each intersection of a column and a subfield are divided into two pairs. If in a pair one BCDN is excited by a receptive field-function originating from one eye, then it will be inhibited by the complementary function, originating from the other eye (see Figure 1.2.2). The other BCDN in the pair is stimulated by the same field-functions, but its inhibitory and excitatory connections are interchanged. The output of one neuron, within a pair, inhibits the other one and causes the rivalry within one depth plane (subfield). This inhibition is such that only one of the two neurons can be active at one time. The other BCDN pair is analogous, but the respective field-functions originate from the opposite eyes. The output of one BCDN also weakly inhibits "neighbouring" BCDNs that may be excited by the other eye. Sperling introduces this inhibition in order to assure that the model could account for the spread of dominance over the contour-free part of the subfield. It is called lateral spread. That it is a weak interaction means only that it is dominated by the other interactions. Thus its influence is present if the other interactions are absent. If an

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are complementary receptive field-functions. If the bars are "wide enough" the contrast between the black bar and its white background also plays a role in rivalry. But according to the visualized explanation (loc. cit. fig. 13) of the phenomenon one has to assume that the complementary function of a function, that describes a black-white contrast, represents a white-black contrast orthogonal - not parallel - to it. This surprising complementarity can be dismissed if one supposes, that the BCDNs indicated in the picture, are neurons whose receptive field-functions describe light or dark fields. However, three types of BCDNs are then involved in the explanation of the phenomenon. Furthermore, it is in conflict with the idea that a receptive field-function is a contour-sensitive function (loc. cit. p. 508). Moreover, the rivalry exists even if the bars are not mutually orthogonal. This means that an almost infinite number of receptive field-functions exists. So, the concept of complementary functions is not a unique one.

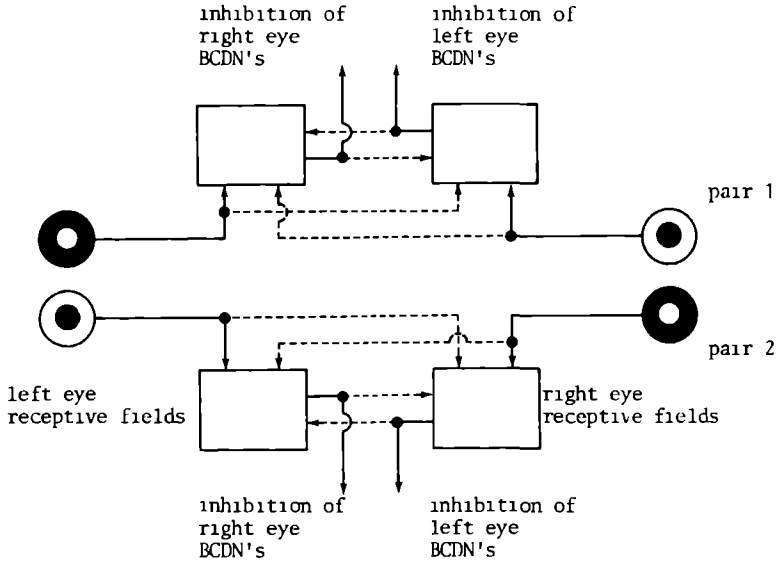

Figure 1.2.2. Intersection of column and subfield (Sperling, 1970).

interaction is not weak it is called strong here.

1.2.1.3. Falsification of the theory of Sperling

With this mechanism Sperling explains a result of an experiment of Levelt (1968) on binocular brightness combination. Levelt (loc. cit. p. 43) shows, that

> "Binocular brightness is constant if the sum of weighted monocular energies is constant; the weighting coefficients are constant for an individual observer,"

but Sperling (loc. cit. p. 509) writes that

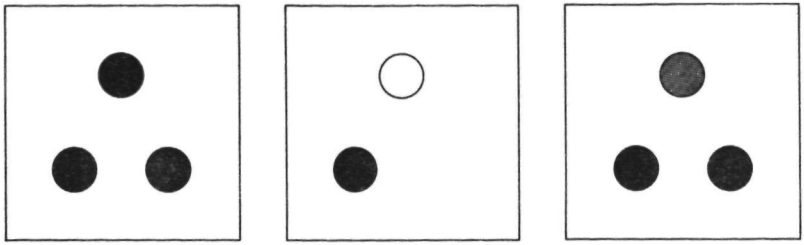

Figure 1.2.3. Brightness mixing (after Levelt, 1968).

> "Levelt summarizes many observations to prove that common contours enable binocular summation in the area within the contours".

According to him the cyclopean image is determined by a combination of the receptive field-functions, that are transmitted by the BCDNs. In the case of rivalry one function is transmitted at a time, but if there is no rivalry the two functions are transmitted simultaneously. In the stimulus used by Levelt (see Figure 1.2.3) the ring and disc have corresponding receptive fields, because they have the same outer contrast. So, both are transmitted. By means of the lateral spread the brightness within both contrast circles is also transmitted. So a dark and a light field are transmitted. Some unknown brightness combination system combines them to a grey cyclopean field.

If one brings some disparity in the ring-disc part of the stimulus (see Figure 1.2.4) the same brightness phenomenon occurs, but the ring-disc combination is perceived in depth. The explanation, according to Sperling, follows straightforwardly from the explanation above. The stimulus pair in Figure 1.2.5 is rivalrous. One is therefore involved with complementary field-functions by definition. If some disparity is brought in this stimulus (see Figure 1.2.6), rivalry is still seen, but depth is also perceived. Following the line of reasoning above, the rivalrous outer contrasts cause this phenomenon. On these grounds one

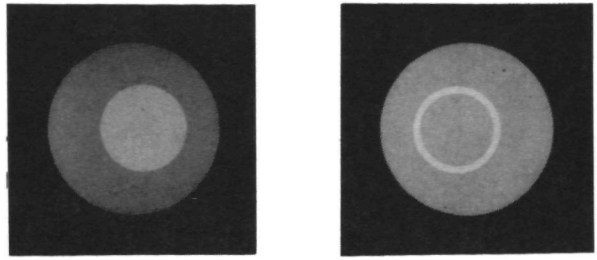

Figure 1.2.4. Depth and brightness mixing.

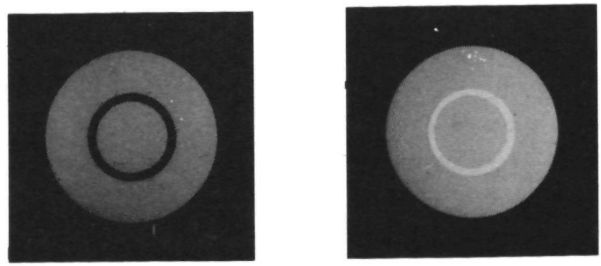

Figure 1.2.5. Rivalry.

would expect that the stimulus in Figure 1.2.7 would evoke the same experience. However, rivalry does occur, but there is no perception of depth (Treisman, 1962; Levelt, 1968) or depth is very difficult to establish (Kaufman & Pitblado, 1965). The difference between the response to the stimulus in Figure 1.2.6 and the response to the stimulus in Figure 1.2.7 can be explained by assuming, that not just the outer-contrast, but the entire contrast plays a role in the binocular combination. The depth perception in Figures 1.2.4 and 1.2.5 may be caused by the common contrast in the ring-disc part of both half-images, as was already known to Helmholtz (1866). That is the outer contrast in both half-images (for Figure 1.2.4) and, for Figure 1.2.5, the outer contrast of one half-image and the inner contrast of the other one. However, in that case the explanation of the brightness phenomenon in Figures 1.2.3 and 1.2.4 is false, because the lateral spread from the disc contrast to the inside would be prevented by the inner contrast of the ring. This problem can be overcome by assuming that the lateral spread

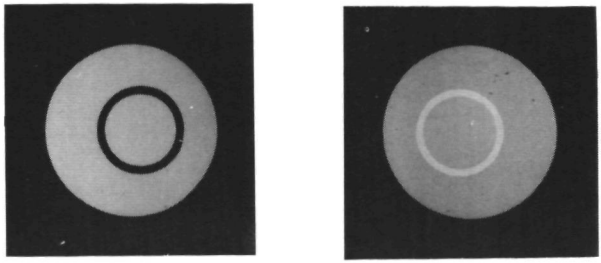

Figure 1.2.6. Depth and rivalry (after Levelt, 1968).

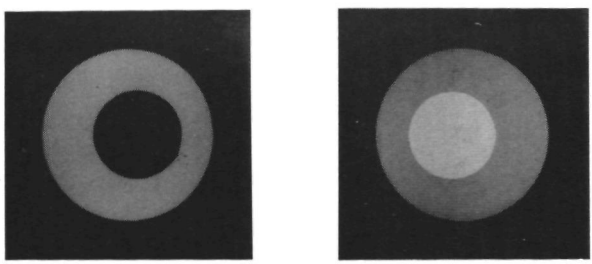

Figure 1.2.7. Rivalry; depth mostly absent (after Levelt, 1968).

is not caused by a weak inhibitory signal, but by a strong one. It will be shown later on that this conjecture is falsified by another of Levelt's experiments. Even the assumption that a disc and a ring are totally different, non-complementary receptive fields cannot explain the difference in the responses to Figures 1.2.6 and 1.2.7, unless one assumes that every type of receptive field-function has a different interaction mechanism in the neural binocular field.

Levelt (1968) did an experiment on lateral spread. The binocular stimulus and the cyclopean image are given in Figure 1.2.8. Levelt measured the dominance of the left field as a function of the disc-diameter. If the diameter is 1 deg of visual angle the left field is 13.7 times stronger in the centre of the disc than the right field. Looking at the table the left field seems not fully dominant, but one can argue that this is due to the calculation method that is based upon his model for binocular brightness combination. According to Sperling's theory about the lateral spread the dominance will not change if the diameter of the

diameter	1 deg	3 deg	5 deg	7 deg
mean[strength left field / strength right field]	13.7	7.9	6.0	5.7

Table 1.2.1. Strength of lateral spread, calculated from Levelt (1968).

disc is changed. However, Levelt's data (See Table 1.2.1) contradict it. One can explain Levelt's results using Sperling's model by assuming that noise interacts with the lateral spread. Therefore the lateral spread is weakened if the disc grows. This explanation is only possible if the noise and the lateral spread are about equally strong. Noise is a weak signal by definition. Thus, this assumption requires that the lateral spread is a weak interaction. But this contradicts the earlier necessary assumption that it is a strong interaction. Consequently noise cannot be the source of the decrease of the lateral spread. One is thus forced to the conclusion that Levelt's experiment shows that lateral spread is not fully self-propagating. But the mechanism of self-propagating lateral spread had to be introduced to explain the spread of dominance. So, Sperling's model is falsified by Levelt's findings, unless one accepts that the interaction mechanism is different for every type of receptive field-function as is stated above. However, such a theory has no strong explanatory power.

Apart from this specific falsification of Sperling's theory objections can be made to the general principles of fusion theories and suppression theories, which will be done below.

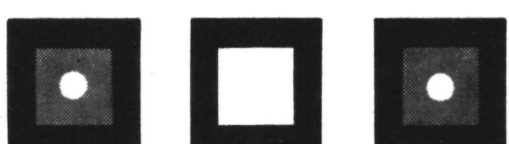

Figure 1.2.8. Levelt's experiment (1968) on field size; both stimuli and the cyclopean image.

1.2.2. General objections against fusion theories

In his discussion of fusion theories of stereopsis Kaufman (1974) remarked that Sperling's theory is a fusion theory in which many elements of suppression theories are incorporated. He states that therefore Sperling's theory has a weak point in common with all fusion theories. It allows the occurrence of displacement to be coupled to fusion. The principle of this mechanism in fusion theories is shown in Figure 1.2.9. If one considers stimuli such as in Figures 1.2.9a and 1.2.9b, then the process of fusion can be represented geometrically, as in Figure 1.2.9c. The filled circles indicate the pattern that is seen. Their distance, measured parallel to the distances of the stimuli, is greater than that of the left stimulus and smaller than that of the right stimulus. As is argued by Kaufman, following Ogle (1964), this displacement can also be established by an imperfect fixation by the perceiver, i.e. if fusion is not guaranteed. The experimental results of Pitblado (1966) support this conjecture, so the displacement will not always be coupled to fusion alone. However, Tschermak-Seysenegg (1952) makes a distinction between sensory fusion, which is what is being discussed here, and motor fusion or vergence. The latter is due to rotations of the eyeball about the vertical axis and regulates the fixation. This distinction is meaningful

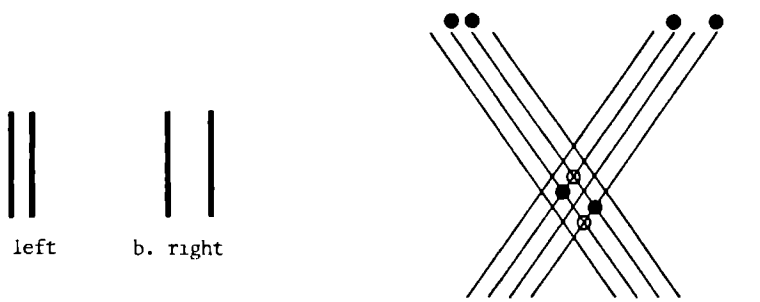

Figure 1.2.9. Schema of the general mecahnism of fusion theories.

because one cannot argue against a theory about sensory fusion on the grounds that it does not incorporate a motor fusion mechanism. Or, to be more precise, the fact that displacement can be caused by a bad fixation does not prove that it cannot be caused by sensory fusion too. Kaufman's general argument cannot be applied to a theory which is solely about sensory fusion. Equally it cannot be applied in a precise way to Sperling's theory as it stands, although Sperling also describes a mechanism for motor fusion. Because, the coupling between both fusion mechanisms is not well defined.

Because in this thesis only sensory fusion is discussed Kaufman's argument cannot be taken as an objection against fusion theories. Another objection of Kaufman to Sperling's theory can be applied equally to other fusion theories. Kaufman (1974) showed that Sperling's theory does not solve the problem of ghost images. If both eyes are stimulated by two vertical black lines on a white surface and if the distances between the two lines in each eye differ a little, two lines are seen in relative depth to each other (filled circles in Figure 1.2.9c). However, four different subfields receive corresponding signals in different columns. So, one would expect to see four lines in relative depth to each other. This is never observed. The two extra lines (open circles in Figure 1.2.9c) are called the ghost images. Krol and Van de Grind (1980) have shown that the "ghost" images can be observed. But then the "real" images are not observed. In fact the "ghost" and "real" images are interchanged. This does not change the argument. This objection is a general on to all theories that use fusion as the basic mechanism for stereopsis **).

Sperling's theory is a theory that introduces local interactions within a column for disparity detection and binocular rivalry. The

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**) Krol and Van de Grind showed that "ghost" images are observed mainly, if the horizontal order of both retinal projections of the objects differ. This is an argument in favour of disparity detection theories, which use retinal projections as the input of a system (see Section 1.2.6).

rivalry interaction is independent of the disparity detection. In order to assure that the theory can account for the global character of binocular rivalry, which typically involves many columns together, he introduces the mechanism of lateral spread mentioned above. A global character is also shown in disparity detection experiments with random dot stereograms. Sperling does not introduce a mechanism for this situation. In all models that solve such stereograms by some point-by-point comparison mechanism, as fusion theories do, many "ghost images" - a multiple of the case in Figure 1.2.9c -, or "false targets" as Julesz (1971) calls them, are evoked by such stimuli. Sperling supposes that his local inhibitory mechanism can solve these ambiguities which normally occur in less sophisticated fusion theories. Julesz (1971), however, shows more generally that local mechanisms cannot deal with these ambiguities in an acceptable way. Here, too, some mechanism of lateral spread is necessary. He demonstrates further that there is phenomenological evidence for this spread. If random dot stereograms with few dots are used, the relatively large regions between the dots in depth are seen in depth too. It means that every theory on stereopsis based on some point-by-point processing model (like fusion theories) needs a form of lateral spread mechanism.

1.2.3. General objections against suppression theories

Helmholtz (1925) showed that the stimulus in Figure 1.2.10 produces a stable cyclopean image, in which the three letters are seen as equally black. It can be understood from Sperling's theory. A and C are both fully dominant. B is seen from both eyes, because the receptive field-functions from both eyes are equal. So, two BCDNs are excited. They are inhibited weakly by each other (lateral spread). Clearly the inhibition does not influence the perception of blackness of one of the letters. The same reasoning holds for the stimuli in Figure 1.2.11. However, now the letters A and C are not seen as black as the letter B, and rivalry can be frequently perceived. During the suppression of C or A, sometimes A, and sometimes C, is seen as equally black as B, but sometimes they

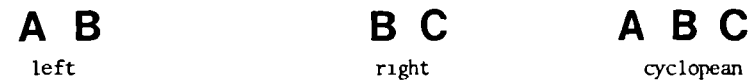


Figure 1.2.10. Stable cyclopean image (after Helmholtz, 1925).

Figure 1.2.11. Rivalrous cyclopean image.

are less black than B. The explanation of the dimmed blackness is, in principle, the same as that of greyness in the ring-disc stimulus in Figure 1.2.3. The rectangles are equal. So equal contour stimuli are involved. However, in Sperling's model rivalry cannot be explained without the assumption that there are complementary receptive field-functions. But complementary field-functions are not involved in Figure 1.2.11. It looks as if a spread of the dominance of C causes a dominance conflict in the contour so that an instability, thus rivalry, occurs. Thus it seems that rivalry can take place by means of the lateral spread. Consequently, the inhibitory mechanism for it must be a strong mechanism. But as we saw from Levelt's experiment in Figure 1.2.8 this assumption leads to an inconsistency in Sperling's theory.

The difficulty of Sperling's theory on this point lies in the all-or-none assumption together with the inhibitory character of the lateral spread. The all-or-none assumption is that only one BCDN in a rivalry pair can be active at any one time. It can be maintained if one drops the inhibitory character of the lateral spread. However, one then abandons the basic idea of suppresion. Therefore this solution will be discussed in the next Section. The fact that Sperling's theory can be falsified is its merit. Because it is the most elaborated suppression theory, it also is the most vulnerable one. Some theories do not even incorporate the existence of lateral spread (Du Tour, 1760; Verhoeff,

1935). However, Levelt's experiment on lateral spread (see Table 1.2.1) showed its existence over more than 1.5 deg of arc. Other theories (Asher, 1953; Kaufman, 1963; Hochberg, 1964) incorporate the fact that suppression does not only occur locally along contrast but also in its neighbourhood. But such a suppression distance does not exceed 7 min of arc (Ogle, 1964; Kaufman, 1963).

A further weak element in all suppression theories is that they cannot account for depth perception (Kaufman, 1974). So many authors (Kaufman, 1974; Nelson, 1975; Sperling, 1970) suppose that the rivalry mechanism and the mechanism for depth perception are different. They suppose that the singleness of vision in the absence of disparity difference between both half-images, or more generally if fusion has been established, is guaranteed by the suppression mechanism. However, such an assertion can be understood perhaps for meaningful stereograms but in the case of random dot stereograms the argument of Julesz (1971) remains valid. He argued that so many ambiguous solutions exist that one needs some lateral spread mechanism, which suppresses ghost images and/or favours the desired solution. Therefore Hochberg (1964) and Levelt (1968) suggested that there is one mechanism for both phenomena. They conjectured that there exists one binocular field. If there is disparity or contrast difference between the two retinal images a cyclopean image, that is not necessarily some direct combination of these images, is sorted out. However, they did not work these thoughts out further, it will be done in Chapter 2.

1.2.4. The permanent rivalry hypothesis

In Sperling's theory the phenomenologically observed suppression during rivalry has been translated into a suppression mechanism at micro-level, the inhibitory character of the lateral spread. However, from the point of view that every cyclopean image is a combination of both half-images (Hochberg, 1964; Levelt, 1968) domination during rivalry is a special case. It can be caused by a suppression mechanism or by a domination mechanism at a micro-level. In the latter case the lateral spread

has an excitatory character. Then in Sperling's model the output of an BCDN would excite similar neighbouring BCDNs. Inhibition or suppression would occur only within one pair of BCDNs. It is called local rivalry. If one eye dominates locally, the system tries to export this domination to the neighbourhood. Within it the other eye can dominate at a different locus. The system also tries to export this domination into the neighbourhood. Thus, normally in a small finite region a part of the loci is dominated by one eye and the remaining part by the other eye. The ratio of the number of members of both parts is determined by the stimuli and the interaction mechanism. It remains constant if one locus in a region changes its domination whilst at the same time a second locus changes its domination in the reversed direction. Thus, even if no rivalry is perceived, it may exist at a micro-level. This principle is called the permanent rivalry hypothesis. It was originally formulated by Levelt (1968) in a somewhat different form. In his formulation the rivalry is not determined locally by an all-or-none mechanism, but rather the cyclopean image is everywhere in the combination of both eyes. The relative strength of the contribution of both eyes is comparable to the ratio mentioned above. Thus both mechanisms or a combination of them are perceptually indistinguishable. Therefore they will be called by one name: the permanent rivalry hypothesis. Thus, the permanent rivalry hypothesis states amongst others that every stimulus pair rivals locally. With this hypothesis Levelt could explain the weighting mechanism he found in binocular brightness combination. Although in this theory suppression is the basis for binocular combination, it is not a classical suppression theory.

1.2.5. Experimental evidence for permanent rivalry

Apart from the fact that for a theory of permanent rivalry the objections made against the classical suppression theories, do not hold, a crucial test between both types of theories has been carried out. Fox and McIntyre (1967) presented pairs of phenomenologically non-rivalrous stimuli. In one of the eyes they flashed a test target that had to be

R-R	F-F	R-F	
1	1	1	F-R
	0	1	R-R
		1	F-F

Table 1.2.2. Theoretical difference in dominance for permanent rivalry and experimental results of Fox and McIntyre (1967).

recognized. The monocular stimulus was a contrast-rich or an almost contrast-free field. According to Levelt (1968) the eye that receives the stimulus with the highest amount of contrast in the neighbourhood of the test target contributes more to the cyclopean image in that part of the field than the other eye. So locally there is a weighted dominance. If both eyes receive the same stimulus there is no dominance, apart from a physiologically determined structural dominance of one of the two eyes. These effects are summarized in Table 1.2.2. An R indicates the contrast-rich and an F the contrast-free stimulus. A pair of R's and F's indicates the two monocular stimuli, one to each eye, that a subject receives. The dominance of the first stimulus of the pair in a column is compared to this dominance in the pair in the row. A one indicates a higher dominance in the column pair, a zero an equal dominance.

Classical suppression theories predict only a suppression of the signal from an eye, if the other eye receives a contrast-rich stimulus. The predictions are summarized in Table 1.2.3a, following the method of Table 1.2.2. The results of the experiments of Fox and McIntyre are in accordance with Table 1.2.2. The differences in dominance are significant ($< .05$), except for the difference between R-F and R-R for one of the three subjects. The results support the permanent rivalry hypothesis against the classical suppression theories, even if one conjectures that some attention mechanisms can play such a role that the test flash is

R-R	F-F	R-F	
0	1	1	F-R
	1	1	R-R
		0	F-F

a. without facilitation

R-R	F-F	R-F	
0	p	1	F-R
	p	1	R-R
		1	F-F

b. with recognition facilitation

Table 1.2.3. Theoretical difference in dominance for classical suppression.

better or worse recognized if there is much contrast in one of both half-images. In the latter case the predictions do not change because the recognition becomes worse for those pairs where rich contrast is present. Recognition facilitation would lead to a prediction as given in Table 1.2.3b. Because the recognition becomes better in those pairs where rich contrast is present there are matrix-cells for which one cannot make a prediction. A letter p is inserted here. It can have the value 0 or 1, depending on the balance between the binocular suppression and the facilitation of the recognition.

One can also make such predictions for fusion theories. They are given in Table 1.2.4. In Table 1.2.4b it is supposed that fusion of identical images facilitates the recognition of the test flash. In Table 1.2.4a this is not assumed. As above, one can suppose that the presence of contrast in the pair facilitates, hampers or does not influence the recognition of the test flash. These situations are expressed by the letter p in Table 1.2.4a. in some matrix cells. It will have the values 1, -1 or 0 respectively. A minus one in a cell means a lower dominance of the first stimulus in the column pair than in the row pair. The value of p in Table 1.2.4b cannot be determined. It depends on the balance between the fusional facilitation and the facilitation or suppression of the recognition by contrast. From Table 1.2.4 it can be concluded that

R-R	F-F	R-F	
0	p	0	F-R
	p	0	R-R
		-p	F-F

a. without facilitation of identical stimuli

R-R	F-F	R-F	
1	1	0	F-R
	0	-1	R-R
		p	F-F

b. with facilitation of identical stimuli

Table 1.2.4. Theoretical difference in dominance for fusion theories.

the results of Fox and McIntyre also falsify the fusion theories.

It is a very important experiment. It was set up as crucial between fusion theories and suppression theories. The conclusion of the authors was that the results fit in better with a suppression theory than with a fusion theory. However, the experiment can be interpreted better as a crucial test between the permanent rivalry hypothesis and the classical suppression theories. A crucial test between fusion theories and suppression theories can be done more easily as Makous and Sanders (1978) showed. These authors conducted an experiment in which they held the contrast in the monocular stimuli equal and constant. By introducing disparity or regular distortion between the monocular field they could introduce depth perception, phenomenological rivalry and normal fusion. They measured the detectability of a test flash that was presented to one of the two eyes. They found no difference in the detectability if fusion or rivalry was perceived. So the suppression is the same in both cases. Thus suppression exists during fusion.

Theoretical considerations and experimental results have led to the permanent rivalry hypothesis. In the next Chapter it will be incorporated into a general theory that also covers the perception of depth. Before doing so, disparity detection theories that try to explain the perception of depth will be discussed.

1.2.6. Disparity detection theories

1.2.6.1. The model of Julesz and a revised formulation

The Julesz model consists of two fields of "dipoles" connected by springs, which cause some co-operative interaction. The dipoles choose a light or dark position in accordance with the local brightness impression. Then an optimal position for all dipoles is sought for by attractive and repulsive forces between dipoles. Dipoles of different eyes attract or repel each other if they have, respectively, an identical or a different light-dark position. These forces decrease with the effective distance between two dipoles. It is called the disparity distance x(n,k) and depends on the orientation of the dipoles as can be seen in Figure 1.2.12. The Julesz model will be formulated below in a more formal way. In this formulation the model survives the counter-argument of Julesz himself, that it does prevent parallel processing. Moreover it is possible to reformulate the formal model into a model with only one dipole field. It is argued in Section 1.2.6.2 that the interaction between input and dipoles in this formulation can be associated with binocular rivalry. A reader who is not interested in the formal description can skip the next two Sections.

In the slightly different, more formal description of the Julesz model, one can represent the light-dark state by some brightness function B of the stimulus intensity I, which has the property that $B(x,y) < 0$, if $I(x,y)$ is perceived as dark, and $B(x,y) > 0$ if $I(x,y)$ is perceived as bright. $|B(x,y)|$ increases if, respectively, the perceived darkness or brightness increases. Attraction between two dipoles occurs if the product b of their B functions is positive (same light-dark states). Repulsion occurs if this product is negative (different light-dark states). For a left image dipole at (nd,md) and a right image dipole at (kd,md) it is given in (1.2.1).

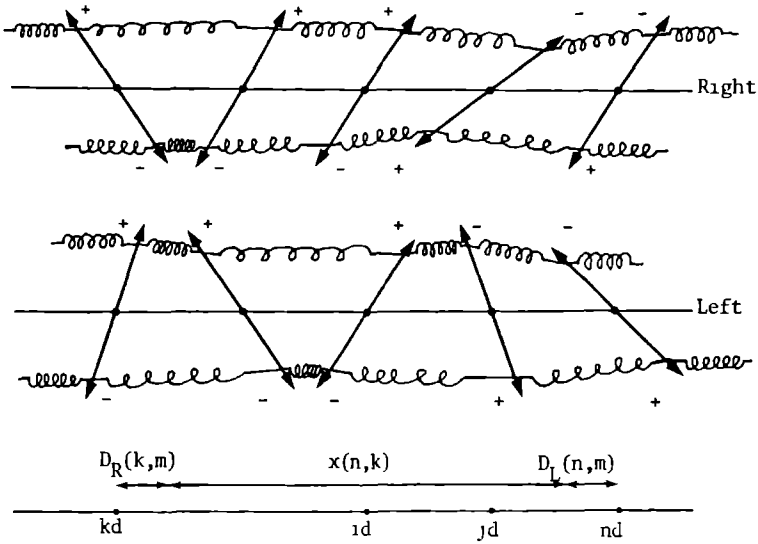


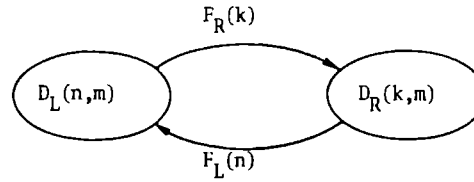
Figure 1.2.12. The Julesz model (1971). + inwards means light position, - inwards means dark position. The left dipole at id and the right dipole at jd are corresponding dipoles.

$$b(n,k) = B_L(nd,md) * B_R(kd,md) \qquad (1.2.1)$$

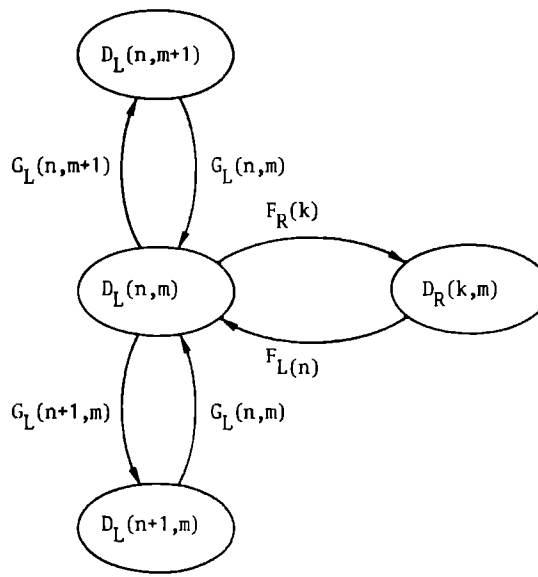
The disparity distance (see Figure 1.2.12) between these two dipoles is equal to

$$x(n,k) = (n-k)d - D_L(n,m) - D_R(k,m) \qquad (1.2.2)$$

The two dipoles are called corresponding dipoles if $x(n,k) = 0$. The influence of the dipole at (kd,md) on the state of the dipole at (nd,md) is described by some function f of $b(n,k)$ and $x(n,k)$. $|f|$ decreases as a function of $|x(n,k)|$. It has the sign of $b(n,k)$. $D_L(n,m)$ is chosen such that $|f|$ reaches its maximum, if $x(n,k) = 0$, i.e. the dipoles at (nd,md) and (kd,md) become corresponding dipoles. However, the dipole at (nd,md) is influenced by all dipoles at (kd,md) for all k. The net-effect of



a



b

Figure 1.2.13. State transition diagram of the Julesz model.

these forces, called the disparity detection input F_L (or F_R), is given in (1.2.3). Now $D_L(n,m)$ is chosen such that F(n) reaches a (local) maximum or minimum.

$$F_L(n) = \sum_{k=-\infty}^{\infty} f(b(n,k),\{x(n,k)\}) \qquad (1.2.3)$$

The system is fairly complicated because a change in $D_L(n,m)$ alters all $F_R(k)$, which moves all right dipoles into another disparity position so that all $F_L(k)$ change. Thus the left dipole at (nd,md) transforms its own input (see Figure 1.2.13a). Such a system can become very unstable. In order to avoid instability another mechanism which prohibits a disastrous influence of one dipole on the whole system is necessary. In the Julesz model the springs (see Figure 1.2.12) provide such a mechanism. It forces an individual dipole to co-operate with its neighbours. The net-force $G_L(n,m)$ for the left dipole at (nd,md) equals zero if the state of the dipole is the mean of its neighbour-state. Otherwise it is forced in the direction of this mean. Let g be some monotonous non-decreasing function with $g(x) = -g(-x)$, the co-operative interaction input $G_L(n,m)$ (or $G_R(n,m)$, see Figure 1.2.13b) can be defined by

$$G_L(n,m) = g(D_L(n+1,m) - D_L(n,m)) + g(D_L(n-1,m) - D_L(n,m))$$
$$+ g(D_L(n,m+1) - D_L(n,m)) + g(D_L(n,m-1) - D_L(n,m)) \qquad (1.2.4)$$

The formal formulation above does not change the gist of the model. There is, however one consequence of the difference with the original presentation of Julesz which ought to be mentioned. In the Julesz' mechanical model the difference between the state value of two neighbouring dipoles could never be such that they "crossed each other". It means that it is impossible (see Figure 1.2.14) for a dipole at point (nd,md) to correspond to one at point (pd,md), i.e. $x(n,p) = 0$, with $p \geq n$, if a dipole at point (kd,md) with $k > n$ corresponds to a dipole at (qd,md) with $q \leq p$. The spring coupling prohibits it. If ambiguous stereograms, in which a depth plane in front of or behind a neutral plane can be perceived, are presented for a brief period, they are usu-

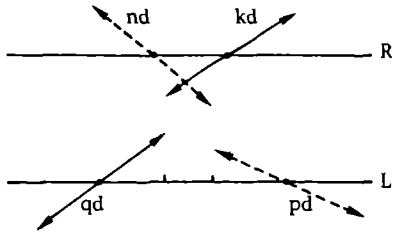


Figure 1.2.14. Crossing dipoles.

ally perceived in one way. So, there is some subject-dependent bias toward one of the two depth levels (Julesz, 1971). Clearly, the subject has a preferred attention for one of the two levels. In the Julesz version of the model which prohibits crossing, it means that all dipoles search for this level together. However, Julesz (loc.cit. p.199, 200) writes that the latter is not plausible because

> "When a slight (10%) bias was introduced (to counteract the subject's natural bias), it proved to be excessive and reversed the perceived depth. If the preferred depth level was the first to be given attention and the other depth levels were only searched sequentially afterwards, then the 90% match at the preferred (natural bias, H.B.) depth level would be more than adequate to stop the search. Instead, the 100% match is perceived at the unpreferred depth level, implying certain models for the attention in stereopsis. One model in agreement with this finding assumes that stereopsis is a parallel process in which each depth plane is simultaneously processed and the one which contains the most activity is attended to."

Simply allowing the fact of "crossing" of neighbouring dipoles in the formulation above does not longer prevent the possibility of parallel processing. The 100% match will give rise to the highest number of

corresponding dipoles at some time, so it will be perceived by means of the co-operative interaction mechanism.

Following Julesz (1971) the perception of relative depth is determined by the difference in disparity between the dipoles in each of the two dipole systems. However, it is not clear from his discussion how the differences in both systems interact. Furthermore, he assumes in all his examples, that the disparity values of two corresponding dipoles are equal. It is easy to prove that the assumption is right if the system has solved the disparity problem. But it does not follow from the model that the assumption holds during the processing stages.

From a psychophysical point of view it does not give rise to difficulties, because during the processing stage no clear image is perceived by subjects. As a consequence many assumptions about the contribution of the two disparity values to the perception of depth are allowed. One can adopt a weaker assumption than the assumption that the disparity values of two corresponding dipoles are also equal during the process. Here the assumption is made that the perception of relative depth is determined by the difference of the arithmetic mean of corresponding dipoles. Roughly spoken, it is assumed that the mean of the differences in the two systems determines the depth perception. This mean equals the differences in each system if corresponding dipoles have the same disparity, which is the case after a solution has been found. Moreover the fact that only the arithmetic mean of the state values of corresponding dipoles, not the values themselves, is important, allows a more simple formal formulation.

In the reformulated model one disparity detector is involved at (pd,md) with internal state D(p,m), instead of two dipoles, one for each image. The mechanisms of this model are similar to the dipole mechanisms described above. For every pair of dipoles $D_L(n,m)$ and $D_R(k,m)$ there is one detector D(p,m) wich can fulfil the requirement

$$D(p,m) = D_L(n,m) = D_R(k,m) \qquad (1.2.5)$$

if $D_L(n,m)$ and $D_R(k,m)$ are corresponding dipoles, i.e. if

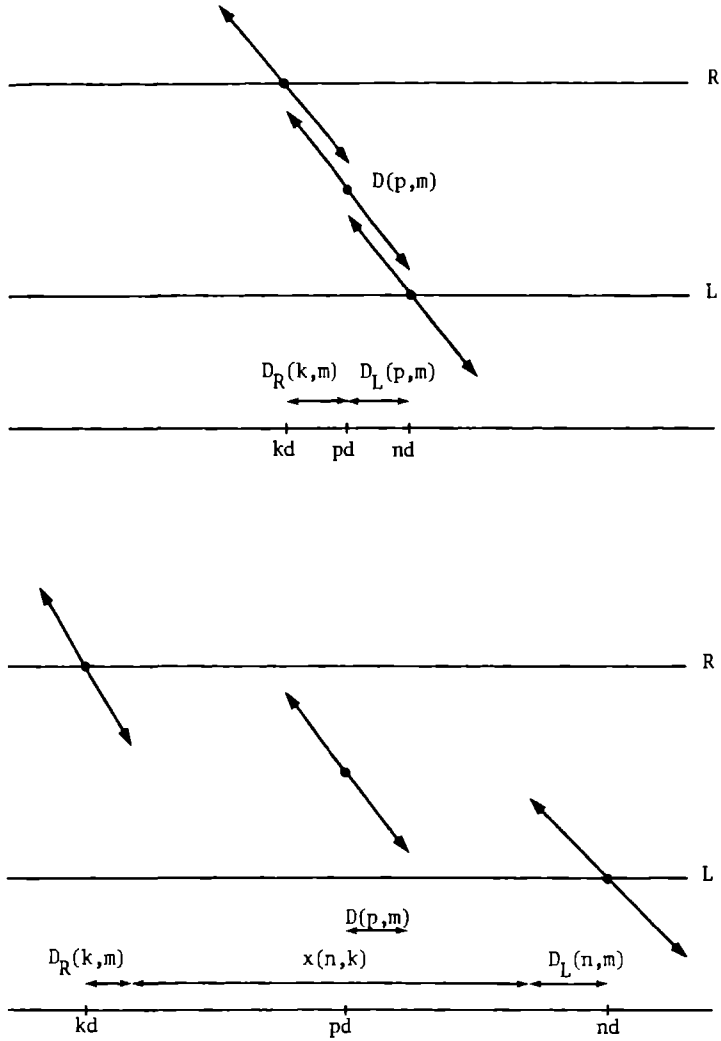


Figure 1.2.15. The relation between the Julesz model and the reformulated Julesz model in the case of corresponding (a), and in the case of non corresponding (b) dipoles.

$$x(n,k) = 0 \tag{1.2.6}$$

after a solution of the disparity problem is found. Then

$$D(p,m) = \frac{D_L(n,m) + D_R(k,m)}{2} = \frac{(n-k)d}{2} \tag{1.2.7}$$

and

$$p = \frac{n+k}{2} \tag{1.2.8}$$

For the new system the solution of the disparity detection problem is the finding of the pair (n,k) for every p, so that (1.2.7) and (1.2.8) are fulfilled (see Figure 1.2.15). The disparity detection mechanism for the detectors can be derived from the disparity detection mechanism for the dipoles if one compares combinations of corresponding dipoles for which holds

$$D(p,m) = \frac{D_L(n,m) + D_R(k,m)}{2} \tag{1.2.9}$$

From (1.2.2) it follows

$$D(p,m) = \frac{(n-k)d - x(n,k)}{2} \tag{1.2.10}$$

with (1.2.8) follows

$$n = p + \frac{x + 2D(p,m)}{2d}$$

$$k = p - \frac{x + 2D(p,m)}{2d}$$

so that (1.2.11) holds for the disparity detection input F(p) (see also (1.2.1) and (1.2.3)).

$$F(p) = \sum_{k=-\infty}^{\infty} f(b(p+[\frac{D(p,m)}{d}]+1,p-[\frac{D(p,m)}{d}]-1),2|1d|) \tag{1.2.11}$$

D(p,m) is chosen such that F(p) reaches a (local) maximum or minimum.

The co-operative interaction input G(p,m) is defined analogously to (1.2.4). The reformulated model is not essentially different from the one above; the state value of the disparity detector is attracted to identical image points and repulsed from non-identical image points; a spatial facilitation of this value exists.

The Julesz model is a great improvement with respect to the fusion theories. It can explain phenomena that are explained by the fusion theories without encountering the difficulty of ghost images. Furthermore, it fulfils the requirement, cited in Section 1.2.2 from the work of Julesz (1971), that every point-by-point processing model requires a lateral spread mechanism, or co-operative interaction.

1.2.6.2. The Julesz model and binocular rivalry

The stereogram in Figure 1.2.16 (after Kaufman et al., 1973) gives rise to binocular rivalry in the presence of depth perception. The depth can easily be explained by the model of Julesz, because the solution of the model with the appropriate disparities is the one that has the highest number of fully locally fused - though only few - image points. Indeed, it is known (Julesz, 1971) that even only a few disparate points can induce a perception of depth. Because of the rivalry one has to conclude that the binocular rivalry mechanism does not precede the

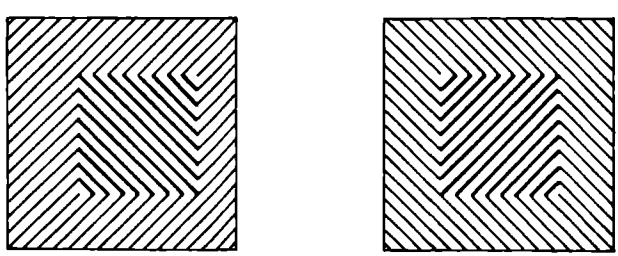


Figure 1.2.16. Rivalry and depth (after Kaufman et al., 1973).

disparity detection mechanism as Julesz introduced it. Otherwise there would not be enough locally fused points to overrule the disparity-rivalry, that is the repulsion, which exists in the model between the white fields and black lines. Here one reaches a weak element in the model. Apart from the non-decisive fact that, phenomenologically, disparity-rivalry is neither perceived here, nor in other stereoscopic stimuli such as in Figure 1.2.4, disparity-rivalry is not a necessary condition to explain stereopsis. The Julesz model would have the same explanatory power, if only identical poles of dipoles would attract each other without any repulsion (rivalry). In the descriptions above it means that in (1.2.1) b(n,k) would be defined to be zero if

$$B_L(nd,md)*B_R(kd,md) \leq 0 \qquad (1.2.12)$$

Furthermore it is less elegant to use the basic feature of binocular rivalry, that is the antagonism between non-compatible brightness levels, in a model to introduce disparity-rivalry without explaining anything about rivalry. It is even more of a problem because binocular rivalry is generally taken (Levelt, 1968) to be what a subject perceives in the non-fusion situation. It is precisely this view on rivalry that is used by Julesz in his model without explaining binocular rivalry. So, in the next Chapter a revised model will be introduced, one that can explain disparity rivalry and binocular rivalry from one principle.

1.2.6.3. The model of Nelson

Nelson (1975) summarizes the arguments of Julesz (1971). He proposes a model in which the interaction mechanism is different from the mechanism in the Julesz model. But the principles of the model are the same. Instead of one disparity detector at the point (nd,md), there are many detectors. Every detector has a constant disparity state value d_j. Two disparity detectors at the same point (nd,md) do not have the same disparity value. All possible disparity state values are present at every point ordered in a kind of column as in the Sperling model (see

Figure 1.2.1). The detectors can be inhibited by the disparity detection input and facilitated by the co-operative interaction input. The disparity detection input for a detector d_i at (nd,md) is the sum of output functions $f(|d_i - d_j|)$ of all other detectors d_j in one stack at (nd,md) which are excited. A detector is excited if (see (1.2.1)) $B_L(nd+d_j,md)$ and $B_R(nd-d_j,md)$ represent points in identical receptive fields. So, if some detector at (nd,md) fuses two points locally, it inhibits all other detectors at that point. Sperling (1970) proposed the same idea in his fusion model (see Section 1.2.1). But the inhibition in the Nelson model decreases, if $|d_i - d_j|$ increases, except in the direct neighbourhood of the detector with state value d_j in the disparity space. The detectors in this region are less inhibited than in a region further from d_j.

The co-operative interaction input of a detector at (nd,md) with state value d_i is the sum of output functions of other detectors with state value d_j. These functions are monotonously decreasing functions of the spatial distance. If this is small, it is a monotonously increasing function of distance. So, there is a spread of every disparity through the spatial domain by facilitation of the detectors with the same state value.

1.2.6.4. A comparison of both models

Kaufman, Bacon and Barroso (1973) constructed random dot stereograms from normal random dot stereograms (Julesz, 1971) in the following way. Both stimuli in the first stereogram are a linear combination of the half-images of the second stereogram. The latter are superimposed, while in one case both coefficients are 1 and in the other case 2-c and c, with $0 \leq c \leq 2$ (see also Figure 6.2.1). From Nelson's model it follows that the interaction process can deliver three perceptually distinguishable solutions. A solution without depth, a solution with a depth plane with respect to a neutral plane and a solution in which the depth is reversed with respect to the other solution with depth. It depends on the value of the variable as to which solution will be chosen. If c tends to zero, one of the solutions with depth will be chosen, if c

tends to 2 the other depth solution is predicted, and the absence of depth can be expected if c approaches 1. For the other values of c the model predicts a rivalry between two of these solutions. The magnitude of the depth in the two predicted cases of depth perception depends only on the disparity in the stereograms. The predictions agree with the findings of Kaufman et al. (op. cit.) for a disparity of 10 min of arc and to a lesser degree for a disparity of 9 min of arc. But they fail for disparities of 8, 6 and 4 min of arc. In the latter case (4 min of arc) the perception of depth is an almost linear function of c. The reason for this falsification is that the disparity detection mechanism is as rigid as Sperling's (1970) fusion mechanism. The stimulus disparity determines which disparity detectors are excited. The excited detectors can silence each other.

In the Julesz model, however, the state value of a disparity detector is attracted to the stimulus disparity, but it will not necessarily reach this value. It depends on the other "attractive forces". If, for instance, c runs from 1 to 2, the model predicts, that firstly no depth will be perceived. Then, if c grows the attraction of the disparate points starts and pulls the state value in the direction of the disparity value. The actual state value is determined by the balance between the two "attractive forces". It tends more and more to the stimulus disparity value as c approaches 2. If the stimulus disparity is so large that the disparity-attraction is almost zero, because it decreases with distance, the model predicts that only two single solutions are possible, one without depth and one with depth. The original Julesz model predicts not only that one of the two is per eived at a given moment, but that rivalry can exist between them. The formulation of the model given here allows the existence of both solutions at the same time, since dipoles may cross each other. Therefore a local rivalry between the solutions is possible. It leads to a clear pattern in depth with some transparent elements, without depth, on it, which is reported by Kaufman et al. (op. cit.). In the model, which will be developed in the next Chapter, the existence of local rivalry follows immediately. In any case one can conclude that the reformulation of the Julesz model is an

improvement because it allows for parallel processing (see Section 1.2.6.1) and the existence of more solutions at the same time.

If the stimulus disparity is not too large, then an important consequence of the fact that the predicted disparity lies between the stimulus disparity and zero is, that the locally fused points do not need to have the same brightness, because they are not the attracting points themselves. Therefore the model predicts that one will not see a clear structure in depth, but some undefinable pulp. The phenomenon is reported by Kaufman et al. (1973). So, in contrast with their remarks, Julesz' model actually predicts their results very well.

1.2.6.5. The existence of other disparity detectors

The notion of receptive field underlies the paper of Nelson (1975). Special neurons or detectors are proposed for many abilities of the visual system. He and other authors - for instance many whose work in spatial frequency analysis will be partly discussed in Section 2.2.1 - rely upon neurophysiological findings which seem to demonstrate the existence of the special detectors. The matter will be discussed in Section 1.2.7. Inspired by these findings Nelson proposed the disparity detectors. But he also proposed the existence of orientation disparity detectors and suggested the existence of spatial frequency disparity detectors. For every type of detector a complete stereoscopic system has to be built up.

It is impossible to conclude from experiments on stereopsis that a spatial frequency detection mechanism exists as long as the "normal" stereopsis mechanism is unknown. If one agrees that some co-operative interaction mechanism and a local disparity detection mechanism are involved, then spatial frequency stimuli stimulate both mechanisms extensively. They activate the local disparity detectors. Their periodicity itself is a global property, conflicting or co-operating with the global effects due to the co-operative interaction mechanism. Hence it can be either in support of or rival the effects induced by the local disparities. Apart from these results all stimuli have a finite size,

which influences the whole mechanism. But frequencies, local disparity and size disparity are coupled quantities. So, one can expect, that even in the case that the "normal" disparity mechanism is known, it will be laborious at best to make any good prediction of its reaction to spatial frequency disparity. This connectivity of several factors just mentioned that influence the mechanisms is shown in Van der Meer (1978).

Nelson claims a proof for the existence of orientation detectors from an experiment by Braddick (Nelson, 1975). According to Nelson (loc. cit. p. 53,54), discussing Figure 1.2.17,

> "The horizontal separation of the bars' endpoints in the top and bottom stereo pair equal, but differ the orientations. If retinal disparity detection is a matter of horizontal displacement only, both stereograms should be processed similarly. Instead, the bars differing more markedly in orientation are perceived as diplopic. The emergence of diplopia as a function of orientation difference when horizontal disparity is constant suggests that stereopsis is here based upon detection of orientation disparity. Braddick's results are contradicted by recent findings of Kertesz (1973). However, Kertesz used lines from 2 deg to 9 deg length, whereas Braddick's stimuli were no longer than 1 deg. The intent of Braddick's study was to reveal cortical single unit properties by employing stimuli compared in size to a cortical receptive field. The stimuli employed by Kertesz were too long. For shorter lines, the visual system's ability to centrally alter perceived orientation probably is limited by the angles involved, not by positional displacement (separation between lines' endpoints)".

However, the latter arguments are only valid if one assumes a priori that the (neurophysiologically found) receptive fields have some psychophysical meaning (see Section 1.2.7). Furthermore, the above results are understandable from the Julesz model. It is known (Fender and Julesz,

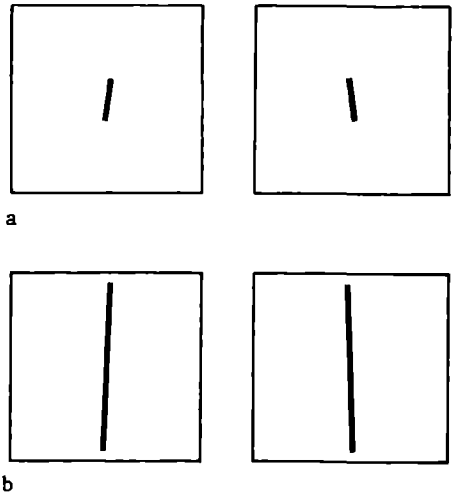


Figure 1.2.17. Rotational disparity (after Nelson, 1975). The notion of fixed disparity leads to special detectors. The notion of flexible disparity and co-operative interaction can explain rotational effects.

1967) that, if eye-movements are prevented, one can pull the stimuli in a stereogram, if they are fused once, slowly apart over a long interval (1 deg - 2 deg) before fusion is lost ***). It is easily explained by the Julesz model, because if the change is not too fast the detectors can follow the movement of the attracting image points. The mechanism shows an analogous reaction to the stimuli in Figure 1.2.17. If two

***) These values hold for horizontal pulling. For vertical pulling fusion is lost at 20 min of arc. This value agrees with values obtained by other authors (Duwaer, 1981, Ch.4.2; Ogle, 1964). Therefore the data on horizontal pulling are probably also reliable. In any case the break away from fusion occurs at greater disparity values than those at which fusion can be attained.

fused points are given, their neighbouring points can be fused even if they don't have the same disparity. The detectors in the neighbourhood of the detectors that connect the fused points are brought into alignment with them by means of co-operative interaction. So they come within the direct influence of the attraction by the neighbouring points. Now it depends on the balance between this attraction and the opposite influence of the co-operative interaction mechanism as to whether fusion will occur or not. Accordingly, as the disparity of the fused points and the disparity of its neighbours that have to be fused grows, this balance changes more and more in favour of the co-operative interaction mechanism. So, fusion can be expected for small orientation disparities as in Figure 1.2.17b and it will break down for greater orientation disparities as in Figure 1.2.17a. Furthermore, a lengthening of the lines supports the stability of the fusion, because there are more co-operating disparity detectors in favour of fusion. Fusion can then be obtained for larger orientation disparities. A further lengthening of the lines can give rise to diplopia when the disparity between the end-points of the lines exceeds the fusional limit (1 deg - 2 deg). So, above some line length, the break-down of fusion is caused by this disparity alone.

1.2.7. Psychophysical theory and neurophysiology

1.2.7.1. General remarks

Psychophysical and neurophysiological knowledge are different types of knowledge about the same object. Neurophysiological knowledge concerns neurons or clusters of neurons. Psychological knowledge concerns large neurophysiological systems, which consist of both (clusters of) neurons as well as the (feedback) relations between them. A model, which is based on neurophysiological knowledge and intends to explain psychophysical knowledge, requires additional assumptions about these rela-

tions.

Such a distinction, between different theories about one type of object, can also be found in other sciences. In physics thermodynamics is a theory about gases. A gas is a set of impinging molecules. The collisions obey Newtonian mechanics. However, one cannot derive thermodynamics from Newtonian mechanics. One needs additional hypotheses about the mechanical interactions in large systems and about the relations between mechanical en thermodynamical quantities.

The difference can be indicated as a difference between a functional and a computational theory. A functional theory is "an abstract formulation of what a system processes and why" (Marr, 1977). A computational theory explains how the system is constructed and why it works on the basis of the given "hardware". Without a computational theory one cannot decide whether a hardware measurement with a functional input is functionally relevant. Therefore one has to be careful of interpreting experimental results which use psychophysical stimuli to produce neurophysiological results. Due to this view, one will not find any decisive argument based on neurophysiological knowledge here. Every fundamental use of it to explain psychophysical phenomena before an adequate formulation of the psychophysics of the problem is doomed to fail. It does not mean that neurophysiological knowledge cannot be fruitful in formulating psychophysical theories. However this has to do with the inspiration of the scientist rather than with his theory.

1.2.7.2. The notion of receptive field

A general remark must be made about the notion of receptive field and more generally about other quantities that are physiologically defined. A receptive field is a technical notion from electrophysiology. It means that some neuron, or cluster of neurons, reacts most strongly to some special stimulus configuration. This configuration is called the receptive field of the neuron. The interpretation of these measurements is another difficulty that arises from using psychophysical stimuli in neurophysiological measurements. One measures local brain activity as a

function of the stimuli. This activity can be due to the properties of the (clusters of) neurons or to the relations between otherwise identical neurons. So without a computational theory one cannot conclude from, for instance, electro-physiological data, that the human brain has special "detectors" for some type of physical stimulus (as has been done frequently from studies analogous to those of Hubel and Wiesel (1962, 1965, 1968), although these authors have always been careful to stay close to a neutral, operational definition of receptive field).

Nelson interprets the results of experiments on receptive fields as a proof for the existence of detectors in the brain for several types of stimuli. Then it appears (see Section 1.2.6.5) that one needs a definition of new detectors and new mechanisms in order to elude psychophysical falsification, since one cannot always explain psychophysical results on the basis of these detectors if other types of stimuli are used. However a theory which incorporates the psychophysics as well as its physiological realisation may not elude psychophysical falsification by introducing hypotheses about the physiology and vice versa. This is a methodological fault.

A psychophysical theory has to be formulated on the basis of psychophysical concepts. One can introduce a psychophysical concept, such as a receptive field-function (Sperling, 1970), and a unit that reacts to it. But one can also introduce more abstract concepts such as the "dipole units" of Julesz (1971). In both cases the relations between the units are psychophysically defined relations. Especially when the relations and units are numerous, as in the models of Nelson and Sperling, it is worthwhile quantifying the theory or comparing it with existing analogous quantitative models, as Julesz did. In any case it is a requirement of scientific methodology to distil the experimental knowledge into a few principles that are psychophysically defined. The only restriction on a psychophysical theory is that the physiological structure must be a possible physical realization, but need not be a unique realization of the theory.

1.2.7.3. Neurophysiology and the presented theory

The mathematical formulation (see Chapter 2) of the cyclopean field and its interactions, which is proposed in this study, allows, in principle, for simulation by some physical device. Details for its construction can be infered from this thesis, but detailed knowledge of the device itself is of no intrinsic importance. However, because it can be done in principle, some analogous mechanisms can exist in the human brain. In fact several such mechanisms would exist and each mechanism would be a realization of the same system, although they would deliver different results in neurophysiological measurements.

It is needless to say that, although concepts like receptive fields are not used in this theory, this does not mean that receptive fields do not exist. It is a functional theory. However, if it makes any sense, it is a guide-line for the construction of a computational theory starting from neurophysiological knowledge. If the concept of receptive field makes sense from a neurophysiological point of view a computational theory must make the bridge between this concept and the functioning of the brain as described in a falsifiable psychophysical theory.

1.3. Permanent rivalry as basis for cyclopean perception

The hypothesis of permanent rivalry has survived crucial experiments in comparison with other theoretical principles. The formulation of the Julesz model presented here can explain the depth phenomena that are due to disparity differences. But the criticism of this model is that it uses the rivalry principle without saying anything about binocular rivalry, while the model would have the same explanatory power if this principle could be disregarded (see Section 1.2.6.2). It is a question of whether rivalry has any linkage to depth perception or not.

Nelson (1975) makes a distinction between the mechanism of depth perception and the rivalry mechanism, analogous to the distinction made by Sperling (1970). Nelson also rules out that binocular suppression can

play a role in the perception of depth. However, he argues against the classical suppression theories. He writes (loc. cit. p. 45) that

> "Rivalry effects, if they occur, are irrelevant to the processes of stereopsis. Stereopsis is based upon the activity of cortical retinal disparity detectors. Units of all disparity tunings save zero would fail to be stimulated by a disparity-free stimulus; rivalrous stimuli might fail to stimulate even this tuning. Because disparity detectors with non-zero tunings are not stimulated, there will be no disparity-domain inhibition. With stimuli which do convey retinal disparity, the present model stipulates that both - and yet neither - of the half-images are suppressed when fusional displacement has occurred. Both half-images are suppressed, in that neither appears where it is physically displayed. Yet neither half-image is suppressed: both contribute to what we see".

This suppression is the inhibition in the disparity space (see Section 1.2.6.3). It has been shown (see Section 1.2.6.4) that this is not an adequate description. On the other hand a mechanism is at work in the Julesz model that resembles the permanent rivalry hypothesis. It is possible that neighbouring detectors have different disparity values. This assumption has been used in the present explanation of the experiments of Kaufman, Bacon and Barroso (1973). It means that in a small finite spatial region a part of the detectors will have one disparity value and another part a different value. The parts are fully intermingled and the ratio between the number of members of the different parts is determined by the strength and the distribution of the different attractive forces and the strength of the co-operative interaction. Because the total number of detectors does not change, one can describe the perceived image in that region as a weighted combination of the different parts whose relative weighting coefficients add up to 1. If the image is psychophysically stable, the coefficients are constant, but by the co-

operative interaction individual detectors can change their state.

This mechanism is exactly the content of the permanent rivalry hypothesis. The hypothesis holds for fused images. If in the Julesz model, the repulsion is dropped and permanent rivalry in the case of fusion is added, we have a system that explains stereopsis and follows the principle of permanent rivalry. Moreover, permanent rivalry can be added without adding an interaction mechanism for it. Its interaction mechanism is, namely, the same as the disparity detection mechanism without the repulsion. So it suffices to transform only the state D(p,m) (see Section 1.2.6.1) into a state-vector (I(p,m), D(p,m)), where I(p,m) can have the values left-image or right-image. The detector keeps its all-or-none character. Thus, the principle of permanent rivalry and the revised disparity detection mechanism in the Julesz model are the natural complement of one other.

As will be shown in the next Chapter the local change in stimulus intensity is the input of the cyclopean system. It can be defined psychophysically in a precise way (see Section 0.1 and Buffart, 1978). In this view brightness and darkness are antagonistic stimuli. Brightness is evoked by increasing the stimulus intensity and darkness by decreasing the stimulus intensity. Monocularly darkness and brightness are rivalrous, simply because brightness is coupled with stimulus on-set and darkness with stimulus off-set. One might argue that much of the phenomenologically perceived binocular rivalry is rivalry between darkness and brightness, so that one would expect it to be the basis for binocular rivalry. However, in contrast with the permanent rivalry during fusion, binocular rivalry is phenomenologically perceived, because the two images are visible for a reasonably long time (Levelt, 1968). This can be explained by the present model. The visibility of a monocular image during binocular rivalry is stable, because permanent rivalry does not disturb it directly. The binocular rivalry between darkness in one half-image and brightness in the other one is intermediated by brightness-darkness rivalry within each half-image and the brightness-brightness and darkness-darkness rivalry (attraction) between both half-images. So, the mechanisms of disparity detection and permanent

rivalry are the natural complements of one other if one generalizes the permanent rivalry hypothesis to monocular rivalry between darkness and brightness. This integrated theory will be presented in Chapter 2.

In this Chapter, which consists of three parts, a formal theory of binocular interaction is developed. In the first part some notions and quantities are introduced and a field-function which describes the values of the quantities is defined. The field-function gives the time-dependent development of the interaction as a function of the stimulus sequences and some systems' interactions. It is argued in part two that this interaction is of a local nature. The implication is that the discussion about the existence of stimulus-oriented detectors in the brain is not relevant. In the third part the consequences of the above-mentioned parts are brought into alignment with the idea that the interaction mechanism is governed by a Markov-process. It leads to a dynamic equation whose stationary solutions predict experimental findings. These are discussed in the next Chapters.

2.1. The notion of cyclopean field

2.1.1. Stimulus location on the retina

Space dependent equations for the cyclopean system make only sense if they can be related to measurable co-ordinates. Therefore special attention is given to the problem of retinal location, since physical distances can only be measured on the retina. Without loss of generality one can describe retinal distances by dimensionless real numbers. A two-dimensional cyclopean space can be defined (Section 2.1.2), which is uniquely related to both dimensionless retinal spaces. Consequently, equations in the cyclopean space can be connected uniquely with measurable co-ordinates and, equally, a dimension can be attributed to distances in the cyclopean space.

2.1.1.1. Space

The projection of a light-intensity distribution onto a retina, called a retinal image, can be described as a luminance-function on a two-dimensional space R^2. From all possible spaces on the retina we regard the R^2, called the retinal space, for which the following holds:

a If the r=(x,y) in R^2 represent the points on the retina then

1 the set of the points with variable x or variable y represent respectively horizontal or vertical lines on the retina.

2 if x > 0, (x,y) is a point to the right of the line x = 0

3 if y > 0, (x,y) is a point above the line y = 0.

b The centre of the fovea is chosen as the origin of the space. Fixation is defined in correspondence with experimental usage, —that is, the image of a fixation point falls on the origin of the space. In practice the definition cannot hold exactly because of drift and saccadic eye-movements (Cornsweet, 1956). But then the "average" place of the images of the fixation point will be the origin.

2.1.1.2. Metrics

Later on (Section 2.1.2) perceptual spaces will be introduced in which a "measurable" unit-length cannot be defined. However, a relationship with "distances" in the retinal spaces can be formulated if their co-ordinates are treated as dimensionless real numbers. This can be done as follows. The distances in the retinal space can be expressed in degree (visual angle) or in millimetre (si-units). If u represents the unit-length in one of these unit-systems, then the physical distance d_{12} between two points (x_1,y_1) and (x_2,y_2) is given by

$$d_{12} = u[(x_1 - x_2)^2 + (y_1 - y_2)^2]^{1/2}$$

So, the x_i and y_i are dimensionless numbers. For every given u the space can be treated as a space on the real numbers. R^2_L and R^2_R are the retinal space of the left and the right eye respectively. L_L and L_R are the luminance distributions assigned to the left and the right eye respectively. The distances on both eyes are expressed in the same units. Two points $r_L = (x_L, y_L)$ in R^2_L and $r_R = (x_R, y_R)$ in R^2_R are called free corresponding points, if $x_L = x_R$ and $y_L = y_R$.

2.1.1.3. Half-images

The part of the physical space that can be observed at time t, is described fully by the two luminance distributions $L_L(r_L,t)$ and $L_R(r_R,t)$. The relation between physical objects in the physical space R^3 and these luminance distributions on retinal spaces R^2 follows straightforwardly from optics, if one allows for eye-lense distortions, blinks and involuntary eye movements. In binocular combination the identity of the shape of these half-images - L_L and L_R - and the disparity between them are very important. The following definitions describe these relations between half-images.

- The monocular stimuli $L_L(r_L,t)$ and $L_R(r_R,t)$ are called identical at time t if for every pair of free corresponding points r_L and r_R it holds, that $L_L(r_L,t) = L_R(r_R,t)$.

- If both half-images differ only in the mean light intensity they are called form-identical, i.e. if there is an $l > 0$, so that $L_L(r_L,t) = lL_R(r_R,t)$ for every pair of free corresponding points.

- Disparity between the half-images can be defined as an opposite translation of a part of both. If for two given stimuli $L_L(r,t)$ and $L_R(r,t)$ and a subset D_L of R^2_L holds that there is a d in R^2, so that $L_L(r-d,t)$ equals $L_R(r+d,t)$ for all r in D_L, then it is said that

for all r in D_L the two monocular stimuli have a disparity magnitude $2|d|$.

- If $L_L(r-d,t)$ equals $lL_R(r+d,t)$ with $l > 0$ then the two stimuli are called form-identical and have a disparity magnitude $2|d|$ for all r in D_L.

This disparity is stimulus-disparity due to relative translation in the half-images. Below (Sections 2.1.3 and 2.1.5) a perceptual quantity, also called disparity, is introduced. Later on (Sections 2.2.3.3 and 2.3.3) the perceptual disparity and the stimulus-disparity are related.

2.1.2. The cyclopean space

It is assumed that the percept is localized in a two-dimensional space R^2: the cyclopean space, denoted by R^2_c. It is a perceptual and not a physical space. It is thus not directly related to physical distance, but it can be related to it by means of definitions. Such definitions can be given consistently by the fact that the notions left, right, above and below are experienced independent of whether the right or the left eye is stimulated, apart from effects that can perhaps be evoked by long term adaptation. Thus, the order that occurs in the retinal spaces is preserved in the percept. In accordance with this an isomorphism $\underline{H}$ can be defined that maps the functions on R^2_L and R^2_R into the set of functions on R^2_c. It is done by introducing two order-preserving mappings H_R and H_L from R^2_c onto R^2_R and R^2_L respectively (see Table 2.1.1). For every function F_L on R^2_c and every function F_R on R^2_c it holds that their mappings under $\underline{H}$, F^c_L and F^c_R, are functions on R^2_c with

$$F^c_L(r,t) = F_L(H_L r,t) \text{ and } F^c_R(r,t) = F_R(H_R r,t).$$

The theory that will be developed below can account for the displacement phenomena that are not due to vergence movements. It is hypothesized here that displacement does not occur in the mapping from the retinal

	left eye	cyclopean space	right eye
space	R^2_L	R^2_c	R^2_R
		$\underset{L}{H}$ <------	------> $\underset{R}{H}$
co-ordinates	$r_L = H_L r$ in R^2_L	r in R^2_c	$r_R = H_R r$ in R^2_R
functions	F_L	F^c_L F^c_R	F_R
		$\underline{H}$ ------>	<------ $\underline{H}$
	$F_L(r_L,t) = F_L(H_L r,t) = F^c_L(r,t)$	$F^c_R(r,t) = F_R(H_R r,t) = F_R(r_R,t)$	

Table 2.1.1. Isomorphism $\underline{H}$.

spaces into the cyclopean space. So it is assumed that H_R and H_L are stimulus independent and equal. Therefore it can be assumed that

$$H_R r = H_L r = r \qquad (2.1.1)$$

2.1.3. The cyclopean field-function

For reasons that will become clear later on three quantities only suffice to describe stereoscopic perception if colourless stimuli are used. They are perceptual quantities that are functions of cyclopean

space and time. The first quantity, called binocular disparity, is a vector $\underline{d}$ in a two-dimensional real number space R^2_d. R^2_d can be conceived of as the set of all values of the perceptual binocular disparity. The second quantity, called signal type, indicates whether darkness or brightness is perceived and whether the left eye or the right eye stimulus is processed. The set of all possible signal-types is denoted by J_c. Thirdly, the perceived strength of the darkness or the brightness is indicated by a non-negative real number. The set of these numbers is denoted by R^+_s.

The values of these quantities as a function of space and time can be described by means of a field-function on the cyclopean space R^2_c, the time-space T, the disparity space R^2_d, the set of signal types J_c and the space of signal-strengths R^+_s. For the moment this last variable will not be considered. The field-function is called the cyclopean field-function $\underline{f}(r,t;d,j)$ with r in R^2_c, t in T, d in R^2_d and j in J_c. It has a unique value - 0 or 1 - for any point and any moment *). It equals 1 if d and j equal the values of the binocular disparity and signal-type at r and t. It equals zero for all other values. The experimental relevance for this formulation will be discussed below, together with the psychophysical character of the sets R^2_d, J_c and R^+_s.

2.1.4. The signal-type, two state phenomena

The set J_c is a set of four elements. These indicate whether the cyclopean field is stimulated by the right or the left eye and whether it is stimulated by a brightness decreasing or a brightness increasing signal (see Section 0.1.3).

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*) In fact $\underline{f}$ is only defined on a finite set of points in R^2_c, but this will be discussed later on.

2.1.4.1. Brightness and Darkness

Gerrits and his colleagues (Gerrits and Timmerman, 1969; Gerrits and Vendrik, 1970a, 1970b, 1972, 1974), among others, have conducted various monocular experiments to look into the so-called filling-in process. A subject is given stimuli stabilized on the retina. When, after stimulus onset, the image is no longer perceived, a part of the stimulus is moved with respect to the retina. Depending on the nature of the movement and the stimulus, brightness and darkness spread over parts of the field.

A L(ight) - D(ark) contrast in the direction of movement (see Figure 2.1.1) results in a darkness spread from this transition in all directions which do not have a component in the direction of the

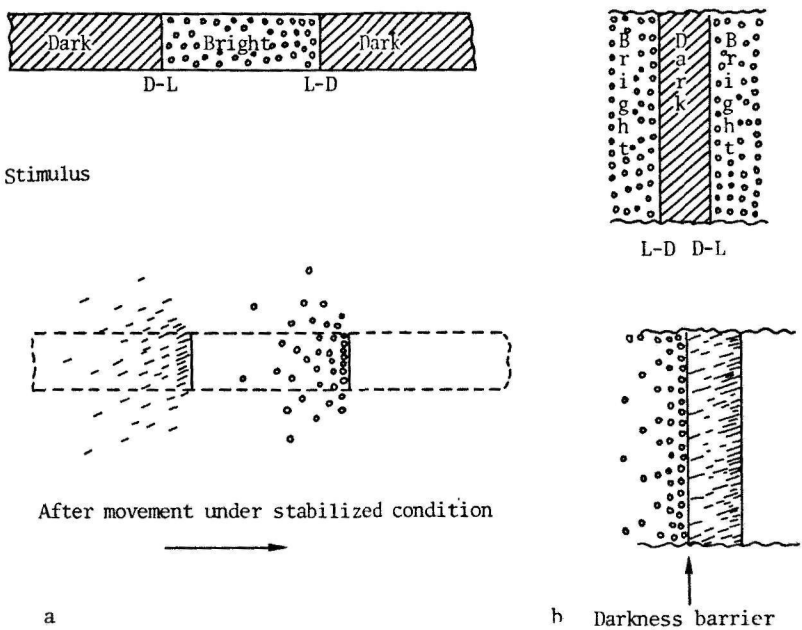

Figure 2.1.1. Darkness and brightness spread by movement under stabilized condition (after Gerrits and Vendrik, 1970).

movement-vector perpendicular to the contrast. For a D-L contrast, the same applies for the brightness spread. Darkness (brightness) never spreads further than an activated D-L (or L-D) transition. The spread is decremental. Gerrits and Vendrik conclude that the cortex has four mechanisms, viz., brightness spread, darkness spread, brightness barrier and darkness barrier, all coupled to stimulus contrast.

The phenomenon of darkness and brightness spread indicates that the change from bright to dark stimulation or dark to bright stimulation of a part of the retina evokes the perception of darkness or brightness respectively. From this it is conjectured that parts of the cyclopean field can be in at least two states: a brightness and a darkness state. From the barrier-phenomenon it is infered **) that a "part of the field" can be only in one state at any one time. It is assumed, that if that part is in a darkness (brightness) state it can transmit only darkness (brightness) signals. On the basis of these assumptions it is understandable that darkness and brightness barriers are perceived. They are caused simply by the fact that a part of the field has been brought into a non-compatible state for the signal that arrives by spreading.

2.1.4.2. Binocular rivalry

Another two-state phenomenon in cyclopean perception is binocular rivalry. As was argued in Section 1.2, during phenomenologically perceived rivalry one almost always perceives, in any part of the cyclopean field, the stimulus of only one of the two eyes. Thus, superposition is

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**) This inference is not the only possible one. One can also suppose that every element combines the two states and that the weighting of the two states in this combination varies from darkness only to brightness only with a continuous range in between (Levelt, 1968). However, for the further development of the psychophysical theory it makes no difference. The theories are equivalent. The precise inference can only be made on neurophysiological grounds. See also Sections 1.2.4 and 1.2.7.

excluded. From this a two-state system for the binocular rivalry - permanent rivalry (see Section 1.2.4) - analogous to that for darkness and brightness is postulated. Similar formulations have been given earlier by Verhoeff (1935), Asher (1953), Hochberg (1964) and Levelt (1968).

2.1.4.3. A four-state system

The exclusion principles do not prohibit the perception of a superposition of darkness and brightness signals or of signals from the different eyes. Later on it will be shown, for instance, that greyness is a superposition of darkness and brightness, and that binocular brightness is a superposition of the brightness of both eyes. But the exclusion principles lay restrictions on these superpositions. The two-state systems can be combined into one four-state system. The usefulness of introducing a four-state system depends on whether the interaction mechanisms that govern the two systems are two separate mechanisms or coupled mechanisms. Some evidence for the latter case will be discussed below. It depends on the character of this coupling how the four states have to be defined. A four state system of the following almost-trivial states is proposed: Brightness-Left, Brightness-Right, Darkness-Left and Darkness-Right. The argument for it is as follows. The existence of perceivable rivalry is doubtful, if the two stimuli in a stereoscopic pair are form identical, as argued in Section 1.2. However, it has been shown that such rivalry is complete if the two patterns in a stereoscopic pair are complementary (Levelt, 1968). In those cases the two patterns have opposite contrast; the light-dark transitions in one half-image are dark-light transitions in the other one, except for some parts, which are identical in order to ensure that the stereoscopic pair will be fused. From this it can be concluded that the binocular interaction between signals of the same type, viz. brightness increasing (brightness) and brightness decreasing (darkness), is different from the interplay between signals of different types. Thus this character of the signal plays a role in binocular interaction. So, the mechanisms of the two-state systems interact. Therefore it makes sense to introduce a

four-state system, whose states reflect the signal types from each retina.

So J_c is the set

$$J_c = \{br, bl, dr, dl\}$$

which means brightness from the right eye, brightness from the left eye, darkness from the right eye and darkness from the left eye respectively. The exclusion-principle ***) is postulated for this system. It means that a part of the cyclopean field can be only in one state at a time. It is the logical combination of the exclusion-principles for the two two-state systems. However, the phenomenological facts that gave rise to these principles support, in the four-state system, only an exclusion-principle for the states that are antagonistic in brightness and darkness. Only the pairs br-dr, bl-dl, br-dl and bl-dr have states which are antagonistic in brightness and darkness. The experimental facts that suggest a generalisation of the exclusion-principle to all pairs of states are the results of experiments on induced rivalry. Makous and Sanders (1978) conducted experiments in which they measured rivalry by means of the objective method (see Section 1.1). They used stimuli with a phenomenologically rivalrous part and with a phenomenologically non-rivalrous part. At first they confirmed the fact (already known) that the dominance periods, reported by the subjects, appear to coincide with dominance periods, measured by the objective method in the rivalrous part of the stimuli. Secondly, they found that these reported dominance periods coincide with dominance periods that are measured by the objective method in some phenomenologically non-rivalrous part of the stimulus. Clearly this rivalry is induced by the rivalrous part of the stimuli because dominance periods are not found if stimuli do not have phenomenologically rivalrous parts. Whatever this induction may be, the fact that these dominance periods can also exist between signals that do not differ in brightness and darkness, but only in origin, give rise to

***) The arguments in footnote **) hold mutatis mutandis here too.

a generalization of the exclusion-principle to these pairs of states. As a consequence the exclusion principle holds not only for the antagonistic pairs (br-dr, bl-dl, br-dl and bl-dr) in which cases it is phenomenologically observed, but also for the equivalent pairs, br-bl and dr-dl in which cases only a suppression-analogy with the rivalrous parts can be measured.

Now, the hypothesis and the postulations concerning the two-state systems can be reduced to two postulates:

a J_c is a set of four states {br, bl, dr, dl}. Between these four elements the rivalry-interactions take place.

b A part of the cyclopean field can be only in one state at the same time.

The definition of the notion "part of the cyclopean field" will be given in Sections 2.3.1.2 and 2.3.2.4. The spring-loaded dipole model of Julesz (1971) is also a local four-state system, apart from the disparity. But the exclusion-principle in the four-state system in that model concerns only the pairs br-dr and bl-dl.

2.1.5. Perceptual binocular disparity

The space of perceptual binocular disparity R^2_d has been introduced, because an observer can fuse form-identical stimuli if the magnitude of the stimulus-disparity 2|d| falls under a certain limit. Fender and Julesz (1967) did binocular experiments with retina-stabilized images in which they moved the stimuli on the two eyes apart. The half-images were form-identical, apart from some cues to depth, and fell upon corresponding retinal places. After the images had been fused the stimuli were moved in opposite directions on the retinas. The general finding was that fusion was lost if the disparity between the stimuli reached a certain critical value. Furthermore, if then the disparity was decreased again, fusion was not immediately regained but only at a systematically

smaller disparity value. However, this latter distance differed from zero. It means that fusion of disparate stimuli can occur; depth can be perceived under these conditions in stabilized vision (Section 1.2.6.5).

Experiments by Evans and Clegg (1967), who evoked stereoscopic depth with afterimages of stereograms, also demonstrate the latter effect. This depth-perception implies detection of relative binocular stimulus-disparity (Julesz, 1971). The magnitude of this depth is a monotonously non-decreasing function (Kaufman, 1973, 1974) of the relative disparity $2d_x$ within some region $2|d_x| < p$ (Panum's area; Werner, 1937; Ogle, 1964). Depth relative to a given plane is defined as positive if an object is perceived farther away from a subject than the neutral plane; it is negative if it is nearer to the subject. The difference in perceptual disparity between two depth planes is called the relative disparity. It is defined positive or negative in accordance with the sign of the relative depth.

2.1.6. Perceived brightness

The stimulus and the interaction mechanism of the cyclopean system determine the time-development of the cyclopean field-function $\underline{f}(r,t;d,j)$. It is assumed that at place r and time t input-signals are transmitted. These signals are the output of the retinal systems. The state d,j of the system at r,t determines which input is transmitted; only those signals can be transmitted which reach r,t and are compatible with the state d,j. The perceived strength at r,t is a combination of all these signal-strengths as for instance their arithmetic mean, their maximum etc. It is assumed that the state at r,t is fully described by d and j. However, it is equally conceivable that the state is given by d,j and s. Where s indicates that only a signal with strength s will be transmitted. Then the perceived signal at r,t is a combination of all local signals with strength s which are compatible with the state d,j. The value of the transmitted signal-strength can change with time. In order to describe it, another field-function which also completely describes s as a function of space and time, has to be introduced. This

cyclopean field-function $\underline{g}(r,t;d,j,s)$ can be defined analogously to $\underline{f}(r,t;d,j)$. A description of the interaction mechanism for the second system can be set up on the same lines. It follows that both systems give the same theoretical predictions except for some brightness matching and estimating experiments. However, just for these experiments an additional assumption has to be made about the procedures which a subject would follow. So, in fact a comparison between the solutions of both mechanisms and the experimental results cannot lead to a decisive choice between them. However, given this assumption the original field-function fits better with these experimental results. So, during the remaining part of the discussion this one will be used. The function $\underline{f}$ can be derived from the function $\underline{g}$ if one assumes the original mechanism. Then the quantity s in $\underline{g}(r,t;d,j,s)$ plays no role in any process at all. It only means that s is the perceived signal strength at r,t. So the function $\underline{f}$ can be defined as

$$\underline{f}(r,t;d,j) = \int_0^\infty \underline{g}(r,t;d,j,s)ds.$$

2.2. The interaction mechanism

2.2.1. The local character of the interaction

2.2.1.1. Global interaction

A priori the construction of the interaction mechanism that transforms the retinal image into the cyclopean image can be based on two different principles. These principles are called global interaction and local interaction.

A stimulus can be described as a combination of more elementary stimuli. It is possible to choose a set of such composing stimuli so

that every stimulus that is not contained in this set can be described almost fully as a unique combination of its elements. For instance color in a stimulus can be described as a combination of three basic colors, and a luminance function on the retina can be described as a linear combination of luminance functions which form a complete orthonormal set.

An analysis of stimuli in terms of such composing elementary stimuli makes sense if the visual system processes its input in terms of those elementary stimuli. Then, in principle, for every such composing stimulus a detector exists. The combination of the output strength of the detectors determines the percept. The detectors are often called channels. One distinguishes, for instance, color-channels (Land, 1964; Graham, 1960) or spatial frequency channels (see below). If the visual system is mainly built up in this way, it is called a mechanism of global interaction, because the elementary stimuli - thus analogously the channels - have been defined on the complete (retinal) space.

Several divergent experimental reports suggest an influence of the global structure of the stimulus on the perception of local elements in the stimulus. Thus Hoekstra, Van der Goot, Van den Brink & Bilsen (1974) found that the visibility of sinusoidal gratings of low spatial frequency (from 1 to 7 cycl/deg) increases with an increase in the number of cycles (from 3 to 8) that are present in the stimulus. Similarly experiments on narrow-band spatial frequency channels (Campbell & Robson, 1968; Graham & Nachmias, 1971; Sachs, Nachmias & Robson, 1971; Graham, 1972; Tolhurst, 1972; De Valois, 1977), on gradient-detection mechanisms (Van der Wildt, Keemink, & Van den Brink, 1976,) and on spatial integration (Thomas, Padilla & Rourke, 1969; Bagrash, Kerr & Thomas, 1971), each seem to support the assumption that some mechanism of global interaction exists.

In fact the only set of orthonormal luminance functions proposed in the literature as a set of composing elementary stimuli, which has been investigated thoroughly, is the set of spatial frequency functions $f(f)=\exp(ir.f)$, where f is the two-dimensional spatial frequency and i is the imaginary number. The choice of this set implies the assumption that the visual system applies some sort of Fourier-analysis on stimuli.

Indeed a lot of experimental results, among which those mentioned above, seem to support the assumption that narrow-band spatial frequency channels exist. However, the results of Henning, Hertz & Broadbent (1975), Stromeyer & Klein (1975) and Wilson & Giese (1977) show evidence against this assumption. Moreover, several authors report contradictory results, for instance on the question of band-width as shown by Stromeyer & Klein (1975). Contradictory results have also been reported on the question of whether the channels act separately. Graham & Nachmias (1971) showed that this is the case, while Stromeyer, Lange & Ganz (1973) found that the channels are sensitive to phase-relations between the frequencies, if they exist at all. To this one can add that experimental results which can be explained in terms of a narrow-band spatial frequency mechanism may also be explained otherwise. The comparison of three different experiments on the visibility of gratings shows that hasty conclusions as to the global nature of interactions are unwarranted.

A first experiment by Van Nes (1968) shows that visibility increases if the mean luminance of the grating increases. At a certain luminance level saturation occurs. Visibility of gratings is defined as the reciprocal of the modulation depth m for a just detectable grating. This last quantity (see Figure 2.2.1) is the quotient of the difference and the sum of the highest and the lowest luminance in a grating

$$m = (L_{max} - L_{min})/(L_{max} + L_{min}) \qquad (2.2.1)$$

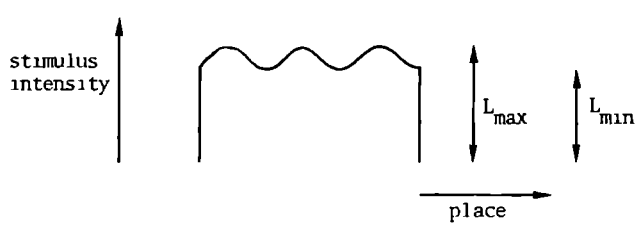

Figure 2.2.1. Modulation depth of a sinusoidal grating.

The result of Van Nes is also corroborated in a second experiment by Hoekstra et al. (1975). Another result of this experiment has been discussed above. This result and that of Van Nes can both be explained with the assumption of narrow-band spatial frequency channels.

In a third experiment by Robson (1966), where the modulation depth has been time-modulated by a sinusoidal function, visibility also increased with the time frequency (up to 10 Hz). This effect can be explained only with the additional assumption that the time characteristic of the channels is a function of their centre-frequency.

However, Buffart (1978) suggested an explanation of the results of these three experiments on the basis of a local (or medium-band) spatial interaction mechanism and a global change of the stimuli due to variations in time. In the experiments of Van Nes and Hoekstra et al. these variations are possibly fully caused by eye-movements. Such an explanation may also hold for the results of the experiment of Graham and Nachmias (1971) mentioned above. They did a threshold detection experiment with two superimposed sinusoidal gratings with frequency ratio 3:1. They concluded that spatial frequency channels exist because they did not find phase-relations. Following their method of calculation these would have been expected if the system is a mechanism of local interaction.

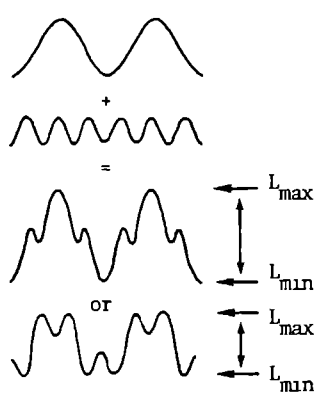

Figure 2.2.2. Addition and substraction of sinusoidal gratings.

Thus their result does not only contradict the result of Stromeyer et al., but also seems to lead to the conclusion that narrow-band channels exist. However, some remarks have to be made about the latter. They supposed in their calculations that the visual system is linear. It is surely not a linear system (see Section 0.1.1). As a measure of the perceived contrast they used the largest peak-to-trough difference (see Figure 2.2.2) divided by their sum analogously to the definition (2.2.1) of the modulation depth in a single grating (see Figure 2.2.1). However, if the retinal system described in Section 0.1 is a good model, the visibility of those stimuli is determined not by the largest peak-to-trough difference but by the largest change in stimulus intensity. This change is caused by eye-movements and stimulus contrast after it has been passed by the non-linear transformation and the retinal mechanism of local interactions.

The general problem in the interpretation of such experiments as decisive between a mechanism of global or local interaction, is that from the latter viewpoint spatially and temporally very complex stimuli are used. Furthermore, eye movements induce a time-dependent variation of a global nature. These problems are particularly present in such direct threshold detection measurements.

There are also experiments on global interaction mechanisms which measure differences in thresholds implicitly due to some adaptation technique. The experimental results in such experiments are the measured differences between two measurements with identical stimuli. But in one of the two cases the subject is adapted to a second stimulus before the measurements start. Thus the aftereffect of the adaptation is used. Extensive reviews of phenomena of aftereffects can be found in Teuber (1960) and Julesz (1971). Another review appears in Kaufman (1974). There are two arguments against conclusions in favour of mechanisms of global interaction based on such experiments. One is that it is not known where adaptation occurs. The other one is the fact that an adaptation theory of a completely local character can explain the results.

Firstly, the findings of global interaction can only be used as an

argument against assumptions of local interaction in the cyclopean system if these phenomena originate from mechanisms before or in the cyclopean system. When an adaptation technique is used to show global interaction, such a conclusion can be drawn only if the adaptation takes place in these first subsystems of the visual system.

Because there is no precise phychophysical knowledge about the place of adaptation mechanisms in the perceptual system, the conjecture that the first processing stages have local interaction mechanisms cannot be falsified by the type of experiment. Furthermore, it is only possible to detect, with the help of stereoscopic techniques, if some adaptation occurs before, in or behind the cyclopean field, if one can prove that the information as to which eye the stimulus originates from, has not been lost in or behind the cyclopean field. Such a proof does not exist. So, with respect to the processing stages, which are relevant for the cyclopean system, nothing can be said about whether adaptation can take place there or not.

The conclusion is that assumptions of local interaction cannot be falsified by experiments that are based on adaptation techniques. Naturally, the same holds true for the conjecture that they have global interaction mechanisms. Consequently the results of such experiments do not add to the degree of confirmation (Popper, 1965) of any of these conjectures since alternative explanations cannot be rejected due to the indeterminacy of the locus of adaptation.

The second argument is as follows. In general aftereffects seem to undo the distortions of normal perception caused by an adapting stimulus. Different types of adaptation such as adaptation to a yellow field (Kohler, 1964), to an up-down reversal (Kohler, 1951, 1953), or to a wide spaced grating (Blakemore & Sutton, 1969) induce corresponding aftereffects such as, respectively, a blue instead of the presented white field, an up-down reversal or a grating that is perceived as more narrow spaced than normal. Some aftereffects can persist for several hours (McCollough, 1965), for several weeks (Stromeyer & Mansfield, 1970; Stromeyer, 1971) even up to four months (Kohler loc. cit.).

The most powerful experiments on the existence of a global

interaction mechanism are the experiments on spatial frequency channels using the adaptation technique. It is used for this purpose by, among others, Blakemore and Campbell (1969), Blakemore & Sutton (1969), Bagrash, Thomas & Kerr (1971), Graham (1972), Tolhurst (1972), Lange and Ganz (1973) and De Valois (1977).

The aftereffect used in the determination of spatial frequency channels is a special case of the so-called figural aftereffects. These are short-term effects and their common feature is that the perception of distances and angles is distorted by the adaptation. The adaptation effects found in all these experiments do not necessitate the conclusion that the visual system is a spatial frequency analyzer. The results can be explained by any theory with the following properties:

1 There exists a two-dimensional perceptual space in which the perception of visual stimuli occurs. It is a Riemann-space.

2 The metrics of this space are influenced by the perceived contrast in the stimulus. Contrast causes a deviation of Euclidean geometry that decreases as a function of the distance of the contrast. Normally, the geometry is Euclidean if there is no contrast perceived. Increasing intensity induces expansion and decreasing intensity induces contraction.

3 The metrics of the empty space are the metrics of the percept of the stimulus, to which a subject is adapted.

4 The metrics act as an attenuation filter.

In Appendix A an example of such a theory which has a completely local character is applied to the adaptation to gratings. Its major result is that the adaptation induces a narrow frequency band that is centered around the adapted frequency. The theory also explains figural aftereffects and geometrical illusions (Watson, 1978).

Finally, an experiment of Enroth-Cugell and Robson 1966 in which the

contrast sensitivity of several retinal ganglion cells of a cat are measured, provides no indication of the existence of a narrow-band frequency mechanism (see loc. cit. Figure 9). Other electrophysiological experiments use, in the main, recordings from the brain (Hubel & Wiesel 1962, 1965, 1968; Campbell, Cleland, Cooper & Enroth-Cugell, 1968). The validity of conclusions from such experiments to the mechanism of the visual system is already discussed in Section 1.2.7.2.

2.2.1.2. Local interaction

In Chapter 1.2 the evidence, that the perception of relative depth that is due to binocular disparity is processed by a mechanism of local interaction has been discussed. The reports of Gerrits and Vendrik (1970a, 1970b, 1972, 1974) on filling-in processes show that local changes in stimulus intensity locally induce the perception of brightness and darkness. If during a perception such changes are suppressed, then this perception fades away (see also Cornsweet, 1966, 1969). Only the local character of the influence of these changes on the percept is emphasized here.

The experiments show directly that an increase or a decrease of the intensity of a stimulus at r in R^2_L or r in R^2_R evokes an increase or a decrease of the perceived brightness at r in R^2_c. The same holds for a neighbourhood of r. But the form and the width of it depends both on the temporal nature of the changes and the magnitude of the intensity variations in the stimulus. Gerrits et al. (1966, 1970b, 1972) showed that a not-too-fast movement of a stabilized retinal image causes a spread of activity which decreases with distance from the contrast border of the image (see Figure 2.1.1). This activity does not spread in the direction of movement. It is perceived as darkness if the change in the retinal stimulation is a decrease in intensity, and it is perceived as brightness if the change in the stimulation is an increase in intensity. These experiments suggest that interactions in the mechanisms that transmit the stimulus from R^2_L or R^2_R into the perceived image on R^2_c are of a local nature. There is no indication in these experiments that the global

structure of the stimuli influences the local activity. The experimental effects occur both for other stimuli such as a disc or a horizontal bar and for other movement-types such as circular and other movements. Phenomenologically the same effects concerned with the brightness and darkness spread occur whatever the form and orientation of the stimuli is. Of course, the form of the spread is co-determined by the form of the stimulus. Even the existence of darkness and brightness barriers which can be observed with faster movements, is concordant with the assumption of a transmission by local activity (see Section 2.1.4.1).

In conclusion, phenomenological observations lead to the idea that the part of the visual system we are interested in consists of mechanisms of local interaction. Seemingly quantitative studies reveal the existence of mechanisms of global interaction: the narrow-band spatial frequency channels. However, starting from a "local" view-point the stimuli used in the latter studies are spatially complex, so that alternative (i.e. local) explanations for the experimental results are not excluded. The results of the adaptation experiments, which directly suggest the existence of narrow-band frequency channels, can be explained on the basis of a local interaction mechanism on their own. There is in fact no conclusive evidence for the existence of global interaction mechanisms. If, moreover, such evidence exists it would be difficult to show that such global interaction were a property of the relatively peripheral subsystems in which we are interested.

The existence of a local interaction mechanism for the detection of binocular disparity has to be concluded from the work of Julesz (1972) with random-dot stereograms. Another explanation could not be found (Julesz, 1971; Nelson, 1975) in spite of many serious attempts. Therefore the findings of narrow-band frequency channels and the related findings in stereopsis have led Julesz (1971) and Nelson (1975) to suggest, that over and above the local interaction mechanism a global interaction mechanism might exist. In Section 1.2.6.5 the weakness of this assumption, as far as stereopsis is concerned, has already been discussed. Thus there is sufficient justification to assume in what

follows that that part of the visual system considered in this thesis is based on mechanisms of local interaction.

2.2.2. The interaction within the cyclopean field

2.2.2.1. The uncertainty of place

It is assumed that the perception of brightness and depth is a kind of measurement: it is an assignment of a characteristic to some area in the cyclopean space. Such an area can be a very small one, but it is never infinitely small. In other words, it is assumed that a subject cannot assign a characteristic to a point in the cyclopean space. If he is instructed to do so, he assigns a characteristic to an area around this point. This assumption should be made because of the existence of different types of noise in the visual system (as, for that matter, in any other type of psychophysical system). Two main sources of noise in the visual system are caused by eye-movements.

One type of eye-movement has been known as tremor. A subject is unaware of tremor and it almost certainly has negligible visual significance (Cornsweet, 1970; Gerrits & Vendrik, 1974). So it can be typified as noise. Its existence means that there is uncertainty in the determination of place in the retinal spaces and thus in the cyclopean space.

A second type of noise consists of the eye-movements, drift and micro-saccades which sustain vision. This sustaining means that the characteristics of a point are determined by the filling-in process in the cyclopean space and the local changes of the light-intensity in the retinal spaces. The latter are due to the eye-movements and the contrast in the stimuli (see Section 2.1.4.1). From the fact that a subject is unaware of these movements, even though they are in principle visible (Gerrits & Vendrik, 1970a, 1970b, 1972, 1974; Moors, Coenen, Gerrits & Vendrik, 1974) it must be concluded that they contribute to the uncertainty of place in the cyclopean space.

This uncertainty means, that a subject always reports about the states of many points collectively in an area in the cyclopean space. If these states are not equal, some criterion is needed for a decision. Later on it will be assumed that the subject's responses are based on the mean of the states. Therefore the number of the points on which the responses are based may be taken, (without any problem) to be finite. This allows for the assumption that the perception is determined by the states of the cyclopean field in discrete points which are homogeneously divided over the cyclopean space.

In order to describe these states the cyclopean detector is introduced. At every point r_1 in the circumscribed area a cyclopean detector is present. A cyclopean detector is a system of which the output and the state are represented by the disparity and the signal type d,j with d in R^2_d and j in J_c (see Figure 2.2.3). The state-value of the detector at r_i reflects the state-value of the cyclopean field at r_1. So $\underline{f}(r_i,t;d,j) = 1$ if and only if the cyclopean detector at r_1, is in the state d,j at time t. $\underline{f}$ is defined only at points r_i which belong to the set of points that determine perception. (cf. the footnote to Section

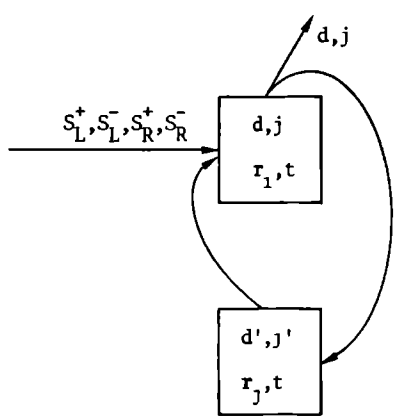

Figure 2.2.3. Cyclopean detector at r_i and its in- output relations.

2.1.3). Let there be N such points. The input of the detector at r_1 consists of the disparity-values and signal-types of other detectors and of the output of both retinal systems which is the input of the cyclopean space at r_i. A cyclopean detector can be regarded as a generalisation of the disparity detector in the reformulated Julesz model in Section 1.2.6.1.

2.2.2.3. The co-operative interaction

As was argued in Section 2.2.1, the interaction in the cyclopean system is of a local nature. The interaction within the system tries to align the state of the detectors over the field, because it spreads the perceived brightness (Gerrits, Haan & Vendrik, 1966; Gerrits & Timmerman, 1969; Gerrits & Vendrik, 1970a, 1970b, 1972, 1974; Moors, Coenen, Gerrits & Vendrik, 1974) and the perceived depth (Julesz, 1971, pp. 121-122 and 199-200; Burt & Julesz, 1980) from one area to neighbouring areas. Therefore it is called co-operative interaction. It is supposed to depend only on the distance between detectors. The local character of the interaction means that the interaction decreases strongly with this distance.

2.2.3. The influence of the input on a cyclopean detector

2.2.3.1. The input of the cyclopean field

Experiments with stabilized images (Cornsweet, 1966, 1969; Gerrits et al., 1966, 1969, 1970a, 1970b, 1972, 1974) have shown that a local change in stimulus intensity evokes the perception of darkness or brightness. The processing that takes place in the retinal system has been formulated in Section 0.1. If L(r,t) represents the luminance distribution on R^2_L or R^2_R at time t, it is transformed into a perceptual field S(r,t).

The reports show that brightness is evoked by an intensity-increase

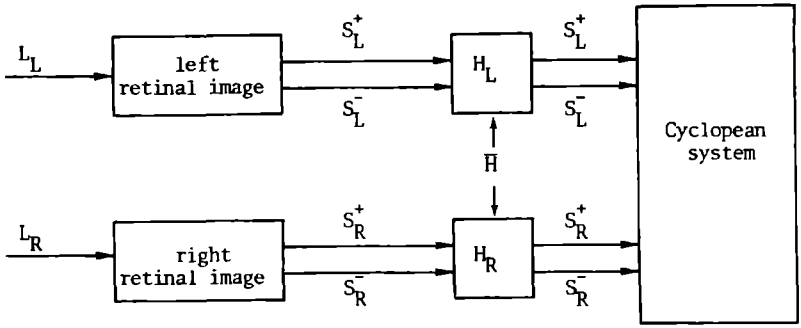

Figure 2.2.4. Input cyclopean system is output retinal system.

and darkness by an intensity-decrease. In Section 0.1 these are represented by $S^+(r,t)$ and $S^-(r,t)$ respectively (see Formula (0.1.11)). It is assumed, that $S_L^+(r,t)$ and $S_R^+(r,t)$ are not only the output of the retinal systems but also by using $H_L r = H_R r = r$ (see (2.1.1)), the input of the cyclopean system (see Figure 2.2.4). Together with the assumption, that there is one cyclopean field, where monocular and binocular interactions take place, this has important consequences for the binocular interaction, as will become clear in Section 2.3.3.

2.2.3.2. The input and the signal type

The changes in the state of the cyclopean detector that are due to some stimulus onset have to be neutralized by the offset of the stimulus. If, for instance, a light onset (left or right) has brought

on	off		
bl	dl	<====>	$w_{bl} S_L^+ - w_{dl} S_L^-$
br	dr	<====>	$w_{br} S_R^+ - w_{dr} S_R^-$

a. states b. weighting coefficients

Table 2.2.1. Bright-dark rivalry.

the cyclopean detector at a point r_i within the stimulated area into a brightness state (bl and br respectively) the offset of the stimulus must be able to change the state into a darkness-state (dl and dr respectively) (see Table 2.2.1a). Later on a probability function for the description of the state of the system will be introduced. It will be assumed that the cyclopean system is a Markovian system. Therefore the probability of the next state is determined by the input and the probability of the actual state of all detectors together. So the probability that an intensity-decrease brings the state of the detector at r_1 from bl (or br) into dl (or dr) does not depend on how the detectors have been brought into the actual state. The systems output - representing a subject's response - is an expectation value of the system. It is some weighted sum of the states of all detectors together. The weighting coefficients are the probabilities of the states (see Table 2.2.1b). So every form of brightness between fully dark and fully bright can be regarded as such a weighted combination of darkness and brightness.

left	right		
bl	-- br	<====>	$w_{bl}\,S^{+}_{L} + w_{br}\,S^{+}_{R}$
dl	-- dr	<====>	$w_{dl}\,S^{-}_{L} + w_{dr}\,S^{-}_{R}$

a. states b. weighting coefficients

Table 2.2.2. Left-right rivalry.

An analogous weighting occurs (De Weert & Levelt, 1974) in the case of binocular brightness mixing. So, if a detector at r_1 is in the state bl it can be brought into the state br by an intensity increase in the right eye (see Table 2.2.2).

However, a darkness decrease in the right eye cannot then bring it into the state dr. Because, if this were possible normally weighted combinations of complementary half-images would be observable analogous to the weighting effects just mentioned in binocular brightness mixing. But complete rivalry is almost always observed (Levelt, 1968, Walker, 1975).

So this phenomenological observation forces one to the conclusion (see also Section 2.1.4.3) that at "microscopic level" - the cyclopean detectors - the binocular interaction takes place only between the equivalent pairs bl-br and dl-dr.

Thus, we arrive at the important conclusion that the phenomenologically observed rivalrous interaction between complementary half-images is absent at the "microscopic level", and that in those cases where no rivalry is observed, rivalry exists at the "microscopic level". This shows the great difference between the permanent rivalry hypothesis, as it is formulated here, and suppression and fusion theories.

Suppression theories suppose a process of rivalry between complementary half-images (Du Tour, 1760; Verhoeff, 1935; Asher, 1953; Hochberg, 1964; Sperling, 1970) and fusion theories suppose the absence of rivalry between form-identical half-images (Boring, 1933; Linksz, 1952; Sperling, 1970).

Permanent rivalry (see Section 1.2.4) is a postulate about the interaction at the "microscopic level". If the detector is in a state br or dr, an increasing or decreasing brightness signal from the left eye

	left	right		
on	bl	br		
	¦	¦	<====>	$w_{bl\,L}\,S^{+} + w_{br\,R}\,S^{+} - w_{dl\,L}\,S^{-} - w_{dr\,R}\,S^{-}$
off	dl	dr		

a. states b. weighting coefficients

Table 2.2.3. Complete rivalry schema.

can bring it into the states bl or dl respectively, while a decreasing or increasing brightness signal from the right eye can bring it into dr or br respectively (see Table 2.2.3). This happens independently of the disparity state-value, because fusion between form-identical disparate stimuli exists (see Section 2.1.5).

Thus, if two form-identical stimuli are stereoscopically fused while their disparity is 2d, all cyclopean detectors that are involved in the fusion have disparity-state value d and rival between left and right. Thus, a detector at r_i is stimulated by the right retinal system output at r_i+d and by the left-retinal system output at r_i-d. The question arises as to whether the cyclopean detector at r_i is also stimulated by the output of the right or left retinal system at an arbitrary place r if it is in some disparity state d. The answer is yes. From a theoretical point of view it is difficult to understand how subjects can easily perceive depth in a random-dot stereogram, if the states of the detectors are not "attracted" towards the disparity that belongs to the depth-perception, because the chance that this disparity is reached randomly is very small (Julesz, 1971). However, there is also experimental evidence for the existence of "attraction". It is discussed in Section 1.2.6.4. There the "attraction" by the output of the retinal system at those places r that do not coincide with $r_i \pm d$ is used explicitly to explain an experimental result of Kaufman, Bacon and Barosso (1973) with the reformulated Julesz model.

So it is assumed that the retinal input of a detector at r_1 that has a disparity state-value d and a signal-type br (dr) consists of all brightness-increasing signals of the output of the left eye. Possibly an output at r is weighted by some distance-function (see Figure 2.2.5) $d(r_1-d-r)$, because the fact that fusion cannot occur spontaneously if the stimulus disparity-magnitude exceeds some value, suggests that $d(r_1-d-r)$ tends to zero if $|r_1-d-r|$ tends to infinity. However, a

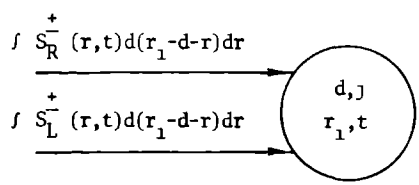

Figure 2.2.5. Input value and disparity state value of a detector at r_i.

discussion about the character of d is not relevant at this moment (see Section 3.1). Furthermore in Section 2.3.3 it is assumed that the retinal output at r tries to change the state d into r_1-r with a "force" that equals the weighted output strength at r. The total force is the sum of all these forces (see Figure 2.2.5). The same holds mutatis mutandis if the original type of the detector is bl (or dl). So the "forces" in the Julesz model and the rivalry "forces" are of the same type. This is the connection between the Julesz model and the notion of permanent rivalry (see Section 1.3).

2.3. A quantitative theory

In this Section a theory about local interactions in the cyclopean space is developed. In Section 2.3.1 a stochastic process for the whole cyclopean space is introduced. The uncertainty of place (Section 2.2.2.1) requires that a percept is related to a class of states rather than to a single state. The contribution of a single state to the percept can be calculated from a probability density function on the state-space, which is derived from a probability on the classes. From this density a stochastic process can be defined. It is supposed that the system as a whole is governed by a Markov process, which means that only the actual state and the input determine the next state.

In Section 2.3.2 the master-equation is formulated. It is a differential form of the (Chapman-Kolmogorov) equation that expresses the Markov character of the system. The solutions of the master-equation can be determined in principle, if the transition probability per unit time is known. It is argued that this probability can be split into a sum of two terms, one due to the co-operative interaction and the other due to other, local, mechanisms. The probability density of finding a detector in a state is defined from the probability density of finding the system in some related state. Then the probability current density between detectors is defined from the co-operative interaction part of the transition probability. The transition probability between the states of one

detector is derived from the other part. At the end of Section 2.3.2 the space variable r is introduced in the equation, so that the original master-equation is converted into an equation on the cyclopean space.

In Section 2.3.3 the probability density currents are formulated such that they fulfil the properties of the interaction mechanism as formulated in Section 2.2. This leads to a set of four coupled differential-integral equations, which is called the systems equation.

2.3.1. A stochastic process

2.3.1.1. Description of the dynamics

In this Section a quantitative theory is developed about the dynamics of the cyclopean system. In order to do that, one needs a quantity which represents the state of the system as a whole. This quantity x represents the state values of all detectors at a time. Let X_1 be the set of states of the detector at r_i - thus X_1 is the set of all values d_{i_1}, j_{i_1} (see Section 2.2.2.2), i.e. $X_i = R_d \times J_c$ - then the set X of states of the whole system is defined as the cartesian product of all sets X_i (see (2.3.1)). Thus x represents $d_{1_1}, J_{1_1}, d_{2_2}, J_{2_2}, \ldots, d_{i_1}, j_{i_1}, \ldots$. If the state value of some detector changes, the value of x changes.

$$X = \prod_{i=1}^{N} X_1 \qquad (2.3.1)$$

Thus the state of the cyclopean system at a time t is defined as the set of state values at time t of all cyclopean detectors. A characteristic function F will be introduced (see (2.3.2)) which is 1 if the state-values which it describes are the actual state-values of all detectors. F is zero otherwise. So F is the expansion of the cyclopean field-function f. It can be defined as the product of all f at a given time, because this product is only 1 if all f are 1 or, in other words, if all f describe the actual state-value correctly.

$$\underline{F}(x,t) = \prod_{i=1}^{N} \underline{f}(r_i,t;d_{j_1},j_1) \qquad (2.3.2)$$

Let the t^J express successive moments, i.e. let $t^k < t^{k+1}$ for all natural numbers k. Let x^J represent the value of x at time $t = t^J$, so that $\underline{F}(x^J,t^J) = 1$ for all t^J. Then the series $x^1,x^2 \ldots,x^i,\ldots,$ reflects the dynamic development of the cyclopean system starting with "initial state" x^1 at the "initial time" $t = t^1$. It is supposed that the dynamic laws that govern this development are independent of the initial state. In other words, it is supposed that the system is characterised by the dynamic laws alone, so that the actual state of the system can be calculated with these laws from the initial state and the history of the input. However, it is difficult to see how one can use the assertions, because noise interacts in an unknown way with the system. Therefore another description of the dynamics of the system, one which incorporates the problem of noise, has to be given. Such a description has a probabilistic character.

2.3.1.2. Probability

The uncertainty of place (Section 2.2.2.1) implies that there are many system states which constitute a percept. Eye-movements are not necessarily the only cause of noise, but this is not important. With noise the state of the cyclopean system fluctuates while the percept remains unchanged. So the state of the cyclopean system at time t cannot be calculated from the percept because a non-denumerable set of states determines the percept. It is possible, however, to relate a percept to a class of states. If the percept is, for instance a very bright field, the cyclopean state belongs to a class of states in which almost all detectors are in a brightness-state rather than to a class of states in which almost all detectors are in a darkness-state. One cannot derive the initial state from the initial percept and if one could do so the presence of noise would make an exact calculation impossible. Therefore it is better to quantify the notion that the system will be found in one

class of states rather than in another class. This can be done by specifying the contribution of each class to the percept and by formulating the laws which govern the development of the preference of the classes. Such a class is called a cluster and may be considered as a perceptual equivalence set. The cluster C(x) of state x is the set of all states x' for which it holds that the percepts of two cyclopean systems which differ only in an interchange of the state x and x', are equal for every time t. It is said that the percept is invariant under the interchange of x and x', or that x and x' are perceptually equivalent. It is not an empty set because x belongs to it by definition. Moreover saccadic movements and drift cause an uncertainty in the disparity of a binocula stimulus (Julesz, 1971, pp. 176-183). So, analogously to the uncertainty of place, it can be concluded that for an arbitrary detector, there is for every disparity d an interval D(d) for which it holds that d and every d' in D(d) can be interchanged without changing the percept. So all states x' which differ from x by such a change in disparity belong to C(x).

In Appendix B the probability r[C(x)], that the states in the cluster C(x) contribute to the percept, is defined. The contribution of such a set to a percept can be calculated from the probability-distribution on the clusters. Let f(x) be some function of the states which has been related to an aspect of the percept. If the probability distribution over the set of clusters can be converted into a probability-distribution over the set of states, then the function f may be regarded as a stochastic variable. Thus the psychophysics of the cyclopean system can be described as a stochastic process and the aspects of a percept, or a perceptual process, can be described as the averages (products) of stochastic-variables.

In order to make calculations over the set, X of all states x rather than over the set of clusters, a probability-density function p(x) has to be defined which fulfils the requirement

$$\int_{y \in C(x)} p(y) = r[C(x)]$$

The notation at the left-hand side stands for a combined summation and integration of the function p (see (B.12)). The integration is carried out over the disparities d_i and the summation is carried out over the signal-types j_i for every detector i. This definition of p(x) has been given in Section B.4 on the basis of the uncertainty of place and disparity. Therefore x changes into a stochastic variable X. Now a family of stochastic variables can be derived from X. A perceptual quantity which is a feature of a percept is defined as the average (product) of stochastic variable(s).

The stochastic variables which are related to X and are also a function of an additional variable t which represents the time, are of special interest. They allow a dynamic description of the system and are called a stochastic process. Every such process F(X,t) has individual functions f(x,t) which describe the mapping from the possible values of X into the possible values of F. f is called the realization of the stochastic process. The probability-density $p_1(y,t)$ that F has the value y at time t is defined by

$$p_1(y,t) = \int_{x \in \mathbf{X}} \delta(y-f(x,t))p(x) \qquad (2.3.3)$$

Straightforwardly $p_n(x^1,t^1;x^2_i,t^2_i;\ldots;x^n_i,t^n_i)$, the joint probability density that X has the value x^i at t^i for i = 1,2,...,n, can be calculated. Then the conditional probability density $q_n(x,t|x^1_i,t^1_i;x^2_i,t^2_i;\ldots;x^n_i,t^n_i)$ that X has the value x at t, if it is known that X has the value x^i at t^i for i = 1,2,...,n can be defined. q_2 and q_1 are defined implicitly in (2.3.4).

$$p_3(x,t;x',t';x'',t'') = q_2(x,t|x',t';x'',t'')p_2(x',t';x'',t'') \qquad (2.3.4a)$$

$$p_2(x,t;x',t') = q_1(x,t|x',t')p_1(x',t') \qquad (2.3.4b)$$

Now the important assumption is made that the stochastic process X is a Markov process. It means that the probability density at t is fully determined by x^1, if X has the value x^1 at succesive moments t^1

with $i = 1,\ldots,n$ and $t > t^1 > \ldots > t^n$. This property is expressed in formula (2.3.5) which holds for all $n \geq 1$.

$$q_1(x,t|x^1,t^1) = q_n(x,t|x^1,t^1;x^2,t^2;\ldots;x^n,t^n) \qquad (2.3.5)$$

Then the conditional probability - $q_1(x,t|x',t')$ -, that X has the value x at t if it has the value x' at t', is called the transition probability. It fulfils the so-called Chapman-Kolmogorov equation (2.3.6a), which follows straightforwardly from the integration over x' in (2.3.4a) after the substitution of (2.3.4b) and (2.3.5).

$$q_1(x,t|x'',t'') = \int_{x' \in X} q_1(x,t|x',t')q_1(x',t'|x'',t'') \qquad (2.3.6a)$$

with $t \geq t' \geq t''$

Thus $p_1(x,t+dt)$ has been fully determined if $p_1(y,t)$ and $q_1(x,t+dt|y,t)$ are known. This is expressed by equation (2.3.6b) which follows from the integration over x" in (2.3.6a) after the multiplication of both sides with $p_1(x'',t'')$.

$$p_1(x,t) = \int_{x' \in X} q_1(x,t|x',t')p_1(x',t') \qquad (2.3.6b)$$

with $t \geq t'$

Equation (2.3.6a) is the basic equation for the dynamics of the cyclopean system, as far as psychophysics is involved. Equation (2.3.6a) can be replaced by equation (2.3.6b). Then one has to look for solutions of the second equation, which fulfil the condition that x=x" if t=t". In Section 2.3.3 the values of q per unit time are infered from experimental knowledge. Before doing so the Chapman-Kolmogorov equation has to be rewritten in the form of a differential quation for $q_1(x,t|x',t')$. The index 1 of p_1 and q_1 will be dropped because joint probabilities do not occur in the discussions below.

2.3.2.1. The master equation

In order to derive a differential equation for p(x,t) it is necessary to know the behavior of q(x,t+dt|y,t) for small dt > 0. In this Section it is supposed that dt $\geq$ 0 and t $\geq$ t'.

Generally the transition probability q(x,t+dt|y,t) can be regarded as the sum of two terms (2.3.7a),

$$q(x,t+dt|y,t) = q'(x,t+dt|y,t) + q''(x,t+dt|y,t) \qquad (2.3.7a)$$

of which q' expresses the probability that during the time-interval (t,t+dt) no transition takes place. Both terms are not negative by definition (2.3.4b), and q' equals zero if x differs from y. Generally it follows for small dt that q' and q" can be rewritten as

$$q'(x,t+dt|y,t) = q'(x,t|y,t) + v(x|y;t)dt + O((dt)^2) \qquad (2.3.7b)$$

$$q''(x,t+dt|y,t) = q''(x,t|y,t) + w(x|y;t)dt + O((dt)^2) \qquad (2.3.7c)$$

If dt = 0 no transition takes place by definition (see (2.3.4b)). Consequently q'(x,t|y,t) has to be regarded as a delta-function of x and y. Since from the definition (2.3.4b) of q it follows that

$$\int_{x\in X} q(x,t+dt|y,t) = 1$$

for all dt, equations (2.3.7d) and (2.3.7e) follow from (2.3.7b) and (2.3.7c).

$$\int_{x\in X} q''(x,t|y,t) = 0 \qquad (2.3.7d)$$

$$\sum_{x\in X} [v(x|y;t) + w(x|y;t)] = 0 \qquad (2.3.7e)$$

$x\in X$

$q''(x,t|y,t) \geq 0$, since $q''(x,t+dt|y,t) \geq 0$ for all dt. Because of (2.3.7d) it holds that $q''(x,t|y,t) = 0$, so that $w(x|y;t) \geq 0$. With (2.3.7e) it follows that the quantity $v_0(y,t)$ which is defined by

$$v_0(y,t) = -\sum_{x\in X} v(x|y;t) = \sum_{x\in X} w(x|y;t) \qquad (2.3.7f)$$

is not negative and can be interpreted as the total probability per unit time for a transition from y to any other value. The quantity $w(x|y;t)$ can be interpreted as the transition probability from y to x per unit time.

It follows from the Chapman-Kolmogorov equation (2.3.6a) that

$$\frac{\partial q(x,t|x',t')}{\partial t} = \lim_{dt\to 0} \frac{q(x,t+dt|x',t') - q(x,t|x',t')}{dt}$$

$$= \lim_{dt\to 0} \frac{1}{dt}\Big[\sum_{y\in X} q(x,t+dt|y,t)q(y,t|x',t') - q(x,t|x',t')\Big]$$

Substitution of the equations (2.3.7a, b, c and f) delivers

$$\frac{\partial q(x,t|x',t')}{\partial t} = \lim_{dt\to 0} \Big\{ \frac{1}{dt}\Big[\sum_{y\in X} q(x,t|y,t)q(y,t|x',t') - q(x,t|x',t')\Big]$$

$$+ \sum_{y\in X} [w(x|y;t)q(y,t|x',t') + v(y|x;t)q(x,t|x',t')]$$

$$+ O(dt) \Big\} \qquad (2.3.8a)$$

$$= \sum_{y\in X} [w(x|y;t)q(y,t|x',t') - w(y|x;t)q(x,t|x',t')]$$

This is the differential form of the Chapman-Kolmogorov equation (2.3.6a). It is fully determined by the transition probability per unit time and the condition that $q(x,t'|x',t')$ equals 1, if x equals x', and equals 0 otherwise. In Section 2.3.3 the transition probability per unit

Figure 2.3.1. State transitions following the master equation.

time is related to the output of the retinal systems. Therefore equation (2.3.8a), serves as a basis for the theory about the cyclopean system. An equation in terms of the probability density function (2.3.8b) can be obtained by an integration of equation (2.3.8a) over x' after multiplying both sides with p(x',t').

$$\frac{\partial p(x,t)}{\partial t} = \int_{y \in X} [w(x|y;t)p(y,t) - w(y|x;t)p(x,t)] \qquad (2.3.8b)$$

Equations (2.3.8) have been known as the master-equation (see Doob, 1953) (see Figure 2.3.1). Since the equations are similar, both can be used to describe the system. For notational symplicity (2.3.8b) will be used. The solutions have to fulfil the condition, that p(x',t') = 1. It is called the initial condition.

2.3.2.2. Cooperative and local interaction

The state x of the system is the state of all detectors (see Formula (2.3.1)). The state of a detector can change by an interaction between neighbouring detectors - the co-operative interaction (see Section 2.2.2.3). It tries to align the state of all detectors and is only a function of the distance between the detectors (see Section 2.3.3.1). State changes of a detector can also be caused by the input of the cyclopean system and an autonomous disparity-changing mechanism in each detector (see Section 2.2.3). These interactions are supposed to depend only on the state of the detectors i.e. on the state of the system. Thus the co-operative interaction is independent of the autonomous disparity mechanism and the input. Consequently the transition probability can be

written as the sum of two independent terms. Similarly it holds for the transition probability per unit time. Let in

$$w(x|y;t) = r(x|y;t) + s(x|y;t) \qquad (2.3.9)$$

r describe the co-operative interaction, then analogously to (2.3.8b) it holds

$$\frac{\partial p(x,t)}{\partial t} = \int_{y \in \mathbf{X}} [\, r(x|y;t)p(y,t) - r(y|x;t)p(x,t) + s(x|y;t)p(y,t) - s(y|x;t)p(x,t) \,] \qquad (2.3.10)$$

Where r is the transition probability per unit time as far as the co-operative interaction is concerned. After the property of locality has been introduced the other term - s - will be partly identified with the change in the retinal output per unit time.

2.3.2.3. Formulation of local changes

The differential equation (2.3.10) can be reformulated as a differential equation which describes the local changes in the cyclopean field, that is for each detector apart. Let $X_i(x_i)$, subset of X, be the set of all states for which the i-th detector is in the state x_i in X_i. The probability-density $P_i(x_i,t)$ over the set X_i - the set of all possible state-values of the i-th detector (see Section 2.3.1.1) - is defined as the integral of the probability-densities on $X_i(x_i)$

$$P_i(x_i,t) = \int_{x \in X_i(x_i)} p(x,t) \qquad (2.3.11)$$

It can be regarded as a probability-density because

$$P_i(x_i,t) \geq 0$$

and with (2.3.11) it follows

$$\int_{x_i \in X_i} P_i(x_i,t) = \int_{x_i \in X_i} \int_{x \in X_i(x_i)} p(x,t) = \int_{x \in X} p(x,t) = 1$$

Let $P_k(x|x_k;t)$ be the conditional probability to find the system in state x at time t if it is given that the state x belongs to $X_k(x_k)$. Thus it is defined (see also (2.3.4)) implicitly by

$$p(x,t) = P_k(x|x_k;t)P_k(x_k,t) \qquad (2.3.12)$$

With this conditional probability and the definition of w (2.3.7) the probability per unit time $-W_{ik}(x_i|x_k;t)$ - for a transition from an arbitrary state of $X_i(x_i)$ into an arbitrary state of $X_k(x_k)$, can be defined (see also Figure 2.3.2).

$$W_{ik}(x_i|x_k;t) = \int_{x \in X_i(x_i)} \int_{y \in X_k(x_k)} w(x|y;t)P_k(y|x_k;t) \qquad (2.3.13)$$

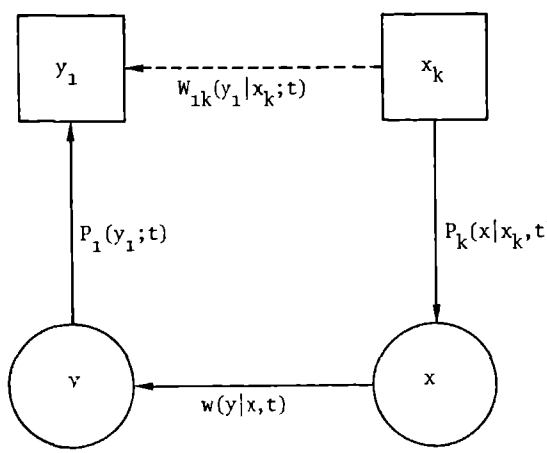

Figure 2.3.2. Derivation of the transition probability of detectors from the transition probability of the whole system.

Analogously, one can derive from $r(x|y;t)$ in (2.3.9) the transition probability R_{ik} per unit time which describes the probability that the i-th detector will be in state x_i if the k-th detector is in state x_k due to the co-operative interaction.

$$R_{ik}(x_i|x_k;t) = \int_{x\in X_i(x_i)} \int_{y\in X_k(x_k)} r(x|y;t)P_k(y|x_k;t) \qquad (2.3.14)$$

The similar transition probability per unit time S_{ik} for the other interactions can be derived from $s(x|y;t)$ in (2.3.9).

$$S_{ik}(x_i|x_k;t) = \int_{x\in X_i(x_i)} \int_{y\in X_k(x_k)} r(x|y;t)P_k(y|x_k;t) \qquad (2.3.15)$$

The co-operative interaction R_{ik} is supposed to be the only interaction between detectors (see Section 2.3.2.2). The interaction S_{ik} is supposed to describe how the state of a detector changes due to the input and the autonomous disparity-mechanism in each detector. It is not an interaction between detectors. Consequently $S_{ik}=0$ if $i\neq k$. Therefore in the derivation of the differential equation for $P_i(x_i,t)$, below, the R_{ik} and S_{ii} are treated separately (see Figure 2.3.3). This derivation is a successive substitution of (2.3.10), (2.3.14), (2.3.15) and (2.3.12) in the first order time-derivative of (2.3.11). Furthermore use is made of the assumption that the number of detectors is denumerable and that the sets of states - X_k - are equal for all detectors k (see Section 2.3.1.1).

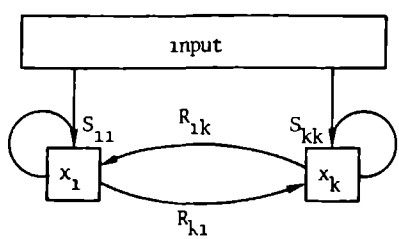

Figure 2.3.3. Co-operative interaction and other interactions.

Therefore the integration-variables over the X_k are replaced by a variable x_0 over the set $X_0 = R_d^2 x J_c$. So an equation (2.3.16) of the dynamics of the system in terms of the individual detectors has been obtained.

$$\frac{\partial}{\partial t} P_1(x_i,t) = \int_{x \in X_i(x_1)} \frac{\partial p(x,t)}{\partial t} \qquad (2.3.16)$$

$$= \int_{x \in X_1(x_i)} \int_{y \in X} [\, r(x|y;t)p(y,t) - r(y|x;t)p(x,t) + s(x|y;t)p(y,t) - s(y|x;t)p(x,t) \,]$$

$$= \frac{1}{N} \sum_{k=1}^{N} \int_{x \in X_1(x_1)} \int_{x_k \in X_k} \int_{y \in X_k(x_k)} [r(x|y;t)p(y,t) - r(y|x;t)p(x,t)]$$

$$+ \int_{x \in X_1(x_i)} \int_{y_i \in X_i} \int_{y \in X_1(y_i)} [s(x|y;t)p(y,t) - s(y|x;t)p(x,t)]$$

$$= \frac{1}{N} \sum_{k=1}^{N} \int_{x_0 \in X_0} [H_{ik}(x_1|x_0;t)P_k(x_0,t) - R_{k1}(x_0|x_1;t)P_1(x_1,t)]$$

$$+ \int_{x_0 \in X_0} [S_{i1}(x_i|x_0;t)P_1(x_0,t) - S_{ii}(x_0|x_i;t)P_i(x_i,t)]$$

2.3.2.4. Expansion of $P_i(x_1,t)$ to the whole space

One can define a function P of the co-ordinates in the cyclopean space - r in R_c^2 - such that it equals the probability density $P_i(x_1,t)$ if $r = r_i$, by

$$P(J_1,d_i,r_1,t) = P_1(x_1,t) \qquad (2.3.17)$$

with $x_i = d_1,J_i$ (see Section 2.3.1.1).

P can be defined such that it fulfils (2.3.17), that r can be treated as a continuous variable and that for every r in R_c^2 it can be regarded as a

probability density on $R_d^2 x J_c$, i.e. as the probability density that an arbitrary point in the cyclopean space will be found in the state d,j. The expansion of $P_i(x_i,t)$ can be based on the definition in Section B.4 of the clusters which are sets of perceptually equivalent states. Such a cluster, which is denoted by C(f(r),d(r),j(r)), is the set of states in which for every r in R_c^2 it holds that the fraction of detectors j in (r,r+dr), for which the disparity d belongs to (d(r),d(r) + dd(r)) and the signal type j equals j(r), equals f(r). For every point r_0 in R_c^2 a set of states $X(j,d,r_0)$, subset of X, can be defined for which it holds that $X(j_i,d_i,r_1) = X_1(x_1)$. Such a set has the property that the state value of the system at r_0 equals d,j surely. It is the conjunction of all clusters with $d(r_0) = d$, $j(r_0) = j$ and $f(r_0) = 1$. Then analogously to (2.3.11) P can be defined for all r in R_c^2 by

$$P(j,d,r,t) = \int_{x \in X(j,d,r)} p(x,t) \qquad (2.3.18)$$

Similarly the functions in the definitions (2.3.12), (2.3.14) and (2.3.15) can be redefined. Thus equation (2.3.16) can be reformulated. It changes into a differential-integral equation (2.3.19). If N in (2.3.16) tends to infinity the summation over k is converted into an integral over R_c^2. The different notation of the transition probability does not require any explanation.

$$\begin{aligned}\frac{\partial}{\partial t}P(j,d,r,t) = \int dd' \sum_{j'} \int dr' [\; & R(j,d,r|j',d',r';t)P(j',d',r',t) \\ & - R(j',d',r'|j,d,r;t)P(j,d,r,t) \;] \\ + \int dd' \sum_{j'} [\; & S(j,d|j',d';r,t)P(j',d',r,t) \\ & - S(j',d'|j,d;r,t)P(j,d,r,t) \;]\end{aligned} \qquad (2.3.19)$$

P(j,d,r,t) can be used to calculate a local aspect of the percept. It can be interpreted as the probability that the percept at r is determined by the output of detectors in the state d,j.

2.3.2.5. Features of the percept

A percept can be characterized locally and temporally by features; a subject reports about the features as a function of place and time. The features can be deduced only from the state of the system which is fully described by the d and j values at every place r. So a feature is a function h of d and j. In terms of the states of the whole system it can be expressed in some kind of characteristic function $\underline{h}$ on the set X(j,d,r). $\underline{h}$ is defined by

$\underline{g}(x|x\in X(j,d,r)) = g(d,j)$ if x in X(j,d,r),
$\underline{g}(x|x\in X(j,d,r)) = 0$ otherwise.

The expectation value of $\underline{h}$ is denoted by <h>[r,t] and is defined by

$$
\begin{aligned}
\langle h\rangle[r,t] &= \int_{x\in X} \underline{h}(x|x\in X(j,d,r))p(x,t) \\
&= \sum_j \int dd\, \Big[\int_{x'\in X(j,d,r)} \underline{h}(x|x\in X(j,d,r))p(x',t)\Big] \\
&= \sum_j \int dd\, \Big[\int_{x'\in X(j,d,r)} h(d,j)p(x',t)\Big] \qquad (2.3.20) \\
&= \sum_j \int dd\, \big[h(d,j) \int_{x'\in X(j,d,r)} p(x',t)\big] \\
&= \sum_j \int dd\, h(d,j)P(j,d,r,t)
\end{aligned}
$$

Indeed, this perceptual quantity is a function of place and time, as was required. So, if the assumption is true that the stochastic process, which can be assigned to the cyclopean system, is a Markov-process, the differential-equation of the probability-densities of local properties contains all information about the system. If R(j,d,r|j',d',r';t) and S(j,d|j',d';r,t) can be formulated, the process of the system is known.

2.3.3.1. Assumptions about the co-operative interaction

As is argued in Section 2.2.2.3, the co-operative interaction aligns the states of the detectors in the field and is a decreasing function of the distance between the detectors. It is assumed that the mechanism does not change with time. So for every detector at r_j the mechanism changes every detector at r_i towards the state d_j,j_j as a function of r_i-r_j and r_j only (see Section 2.3.2.2). Thus the transition function R as it is defined in (2.3.14) and (2.3.19) is a function of r-r' and r' and is zero if $j \neq j'$ or $d \neq d'$ (see (2.3.21)). In other words, it increases the probability-density $P(j_j,d_j,r_i,t)$, which shows the co-operative character.

$$R(j,d,r|j',d',r';t) = R'(r-r',r') \text{ if } j = j' \text{ and } d = d',$$
$$R(j,d,r|j',d',r';t) = 0 \text{ otherwise.} \qquad (2.3.21)$$

where R' is a monotonically decreasing function of r-r'. The index of R' will be dropped below.

The transition probability per unit time, R in (2.3.21), is a probability density current. Therefore (2.3.21) has to be read as follows. If the sy tem at r' is in state d',j' this current tries to swell the probability density that the system at r is in state d',j' - p(j',d',r,t) - as a function of r-r' (and r'). Because of the conservation of probability the probability density that the system at r' is in another state, for instance d,j, can decrease. However, the other transition probability, S (see below), governs the probability density current between d',j' and d,j. It is a function of d-d'. Thus the fact that R equals zero, if $d \neq d'$ or $j \neq j'$, does not mean that the interaction within the cyclopean space, between detectors, is independent of d and d', and of j and j'. One could say that the effort of one detector to "persuade" another detector into the same state is independent of the states

itself. It depends only on the distance between the detectors. However, the effect of the effort does also depend on their states.

2.3.3.2. Assumptions about the other interactions

It is assumed that S does not depend on time explicitly and that it is spatially homogeneous. The first assumption proposes that the character of rivalry and disparity interaction within a detector does not change with time. The second is based on the identity of all detectors (see Sections 2.2.2.2 and 2.3.1.1). It is further assumed that S is the sum of two independent parts S_1 and S_2. S_1 describes an autonomous disparity interaction mechanism and S_2 describes the input influence on a detector.

With respect to the autonomous diparity-mechanism the following considerations hold. If two monocular stimuli in a stereogram are pulled apart (Fender & Julesz, 1967) not only can fusion be maintained if the speed of this shift is not too fast (see Section 1.2.6.5), but also the disparity tends to zero if the stimulation ends at some moment. This experiment yields two conclusions. Firstly the transition probability per unit time is a function of **d-d'**, which tends to zero if |**d-d'**| tends to infinity. Secondly there is an input independent mechanism which forces the disparity of a detector to become zero. Which means that it is also a function of **d'** and that a transition to **d** if **d** lies nearer to the origin than **d'**, is more likely then if it lies farther away from the origin. Moreover S_1 equals zero if j does not equal j', since S_1 is supposed to be stimulus independent and because only the input contribution to S_2 causes the rivalrous transitions (see Sections 2.1.4 and 2.2.3.2). Thus S_1 can be rewritten as

$$S_1(j,d|j',d';r,t) = T(d-d',d') \text{ if } j = j',$$
$$S_1(j,d|j',d';r,t) = 0 \text{ otherwise.} \qquad (2.3.22)$$

where T is a monotonously decreasing function of |**d-d'**|.

With respect to the stimulus dependent part the following

considerations hold. The disparity d of a detector has been introduced to account for the fact that two form-identical disparate stimuli can be fused. This fusion means (see Section 2.2.3.3) that the input from the left eye at r in R_c^2 is transmitted by every detector j at r if $r_j - d_j = r$, and the one from the right eye at r' in R_c^2 by every detector i at r_i if $r_i + d_i = r'$. In these cases it is as if the distance between the detector-place and the input-place is zero. Therefore it is assumed that such an apparent distance (see also Formula (1.2.11)) between a detector j and the input from the left or right eye at r is a function of, respectively, $r_j - d_j - r$ and $r_j + d_j - r$.

It has been argued in Section 2.2.3.2 that if a detector is in some state it can change into another state with which it forms a rivalrous pair by means of an input which is of the same type as the second state. Other input-dependent transitions do not occur. The transition-probability S per unit time (see Section 2.3.2.3) has been formulated as a transition-probability for one detector on the basis of the assumptions which are made in Section 2.2.3.2. These lay two constraints upon the formulation of S_2, the input-dependent part of S. Firstly, j and j' in $S_2(j,d|j',d';r,t)$ comprise a rivalrous pair and secondly, only that input-part is incorporated in S_2 which is of the same type as j. Thus S_2 is unequal to zero only if j and j' form one of the pairs (bl,br), (br,bl), (dl,dr), (dr,dl) or one of the pairs (bl,dl), (br,dr), (dl,bl), (dr,br). For both series the input types which are incorporated in S_2 are respectively S_L^+, S_R^+, S_L^-, S_R^-.

In Section 1.2.6 arguments are given for the property of the Julesz model that the interaction between the input and the detector k is a decreasing function of the apparent distance between detector and input. It has been assumed implicitly that the cyclopean space is homogeneous as far as this interaction is concerned. In the description used here, it means that this interaction is a function of $r_k - d_k - r$ or $r_k + d_k - r$ for the input at r from the left or right eye respectively. Thus, if in $S_2(j,d|j',d';r,t)$ j and j' form a rivalrous pair and if the input-type at r-d, casu quo r+d, is of type j then the input influence depends only on this input and on the apparent distance (see Figure 2.3.4). The lat-

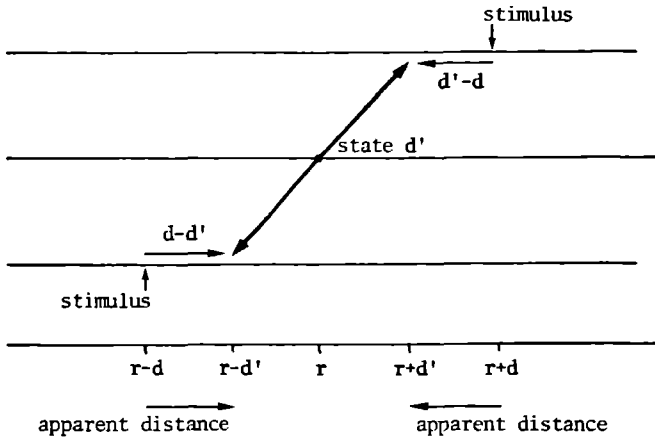

Figure 2.3.4. Apparent distance between a detector at r in state d' and the input at r-d from the left retinal system or at r+d from the right retinal system.

ter is a function of the disparity-difference $\pm(d-d')$. Thus the input influence tends to zero if $|d-d'|$ tends to infinity.

With the assumptions that S does not explicitly depend on time, but that it is time-dependent as far as the input changes with time, and that it is spatially homogeneous, it follows from these considerations

that S_2 can be formulated for binocular rivalry as (2.3.23a)

$$S_2(bl,d|br,d';r,t) = S'(S_L^+(r+d,t),d-d')$$

$$S_2(br,d|bl,d';r,t) = S'(S_R^+(r-d,t),d'-d)$$

$$S_2(dl,d|dr,d';r,t) = S'(S_L^-(r+d,t),d-d')$$

$$S_2(dr,d|dl,d';r,t) = S'(S_R^-(r-d,t),d'-d)$$

and for darkness-brightness monocular rivalry as (2.3.23b)

$$S_2(bl,d|dl,d';r,t) = S''(S_L^{+}(r+d,t),d-d')$$
$$S_2(br,d|dr,d';r,t) = S''(S_R^{+}(r-d,t),d'-d)$$
$$S_2(dl,d|bl,d';r,t) = S''(S_L^{-}(r+d,t),d-d')$$
$$S_2(dr,d|br,d';r,t) = S''(S_R^{-}(r-d,t),d'-d)$$

$S_2 = 0$ in all other cases. (2.3.23c)

2.3.3.3. The systems equation

The functions $S_R^{+}(r,t)$ and $S_L^{+}(r,t)$ are non-linear transformations of the stimulus intensity-distribution function. It is supposed that all non-linearities, as far as the input is concerned, are included in the S^{+} functions. Thus it is supposed that S' and S'' are linear functions of the S^{+}.

Consequently S'(F,d) and S''(F,d) can be rewritten as

$$S'(F,d) = Ff(d)$$
$$S''(F,d) = Fg(d) \quad (2.3.24)$$

From Section 2.3.3.2 it follows that f and g tend to zero if |d| tends to infinity. The characteristics of these functions cannot be esta-

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See opposite page:

Figure 2.3.5. The rivalry equation at two points r and r'. Every point (detector) has an input selected from both retinal outputs. The autonomous disparity interaction T and this input change its state value only. R represent the co-operative interaction.

T(d'-d,d)
$S_L(r_m)$
$S_L(r_n)$
$r_i = r+d'$
f(d-d')S_L(r+d')
d,j
f(d-d')S_R(r-d')
$r_i = r-d'$
$S_R(r_m)$
$S_R(r_n)$
R(r'-r,r)
R(r-r',r')
$r_j = r'+d'$
d,j
$r_j = r'-d'$
f(d-d')S_L(r'+d')
f(d-d')S_R(r'-d')
T(d'-d,d)

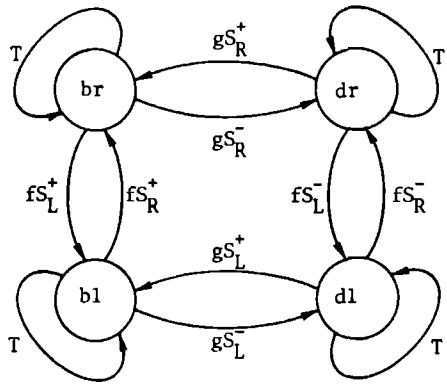


Figure 2.3.6. The complete rivalry-transition schema at a point of R^2_c (detector) analogous with Table 2.2.3a.

blished exactly. But from the literature (see for instance Julesz, 1971, chapt. 5) it can be infered (see Section 3.1.1) that f and g are symmetrical in d_x and d_x'. Similarly it can be assumed that $T(d,d')$ is symmetrical in (d_x,d_x') and (d_y,d_y'). Furthermore the disparity-interaction is phenomenologically a binocular interaction. So g probably tends to zero faster than f if $|d|$ tends to infinity, because f and g are both functions of the apparent distance (see Section 2.3.3.2).

The systems equation (2.3.25) follows after substitution of (2.3.21), (2.3.22), (2.3.23) and (2.3.24) in (2.3.19). See also Figures 2.3.5 and 2.3.6 and Table 2.2.3. The first four terms in equation (2.3.25) describe the co-operative interaction, the second four the autonomous disparity-mechanism and the other two groups represent the rivalrous interactions due to the input. The first of these two is the binocular interaction and the second one the monocular, or brightness-darkness interaction. They are based on the principle of permanent rivalry (see Sections 1.2.4 and 1.3). Therefore this set of coupled differential-integral equations is also called the rivalry theory of the cyclopean system and the formal system is also called the rivalry system.

This rivalry theory serves as a basis for an explanation of the

perception of brightness (Chapter 4), disparity-detection (Chapter 5) and binocular rivalry (Chapter 6). Spatio-temporal interactions can also be described (Buffart, 1978). In Chapter 3 it is shown that the theory fulfils some basic requirements. These explanations are based on stationary solutions.

$$\frac{\partial}{\partial t}\begin{bmatrix} P(br,d,r,t) \\ P(dl,d,r,t) \\ P(dr,d,r,t) \\ P(bl,d,r,t) \end{bmatrix} = \int dr' \{ R(r-r',r') \begin{bmatrix} P(br,d,r',t) \\ P(dl,d,r',t) \\ P(dr,d,r',t) \\ P(bl,d,r',t) \end{bmatrix} - R(r'-r,r) \begin{bmatrix} P(br,d,r,t) \\ P(dl,d,r,t) \\ P(dr,d,r,t) \\ P(bl,d,r,t) \end{bmatrix} \}$$

$$+ \int dd' \{ T(d-d',d') \begin{bmatrix} P(br,d',r,t) \\ P(dl,d',r,t) \\ P(dr,d',r,t) \\ P(bl,d',r,t) \end{bmatrix} - T(d'-d,d) \begin{bmatrix} P(br,d,r,t) \\ P(dl,d,r,t) \\ P(dr,d,r,t) \\ P(bl,d,r,t) \end{bmatrix} \}$$

$$+ \int dd' f(d'-d) \begin{bmatrix} S_R^+(r-d,t)P(bl,d',r,t) - S_L^+(r+d',t)P(br,d,r,t) \\ S_L^-(r+d,t)P(dr,d',r,t) - S_R^-(r-d',t)P(dl,d,r,t) \\ S_R^-(r-d,t)P(dl,d',r,t) - S_L^-(r+d',t)P(dr,d,r,t) \\ S_L^+(r+d,t)P(br,d',r,t) - S_R^+(r-d',t)P(bl,d,r,t) \end{bmatrix}$$

$$+ \int dd' g(d'-d) \begin{bmatrix} S_R^+(r-d,t)P(dr,d',r,t) - S_R^-(r-d',t)P(br,d,r,t) \\ S_L^-(r+d,t)P(bl,d',r,t) - S_L^+(r+d',t)P(dl,d,r,t) \\ S_R^-(r-d,t)P(br,d',r,t) - S_R^+(r-d',t)P(dr,d,r,t) \\ S_L^+(r+d,t)P(dl,d',r,t) - S_L^-(r+d',t)P(bl,d,r,t) \end{bmatrix}$$

(2.3.25)

In this Chapter it will be shown that the general theory can explain certain phenomena which every theory on cyclopean perception has to explain. First, the character of the functions f, T and R (see (2.3.25)) is discussed. Second, the behaviour of the system, if t tends to infinity under special stimulus conditions, is analysed. Finally, some remarks about spatio-temporal processes are made.

3.1. Remarks about the functions f, T, and R

3.1.1. The range of the function f

The systems equation has transition probabilities which are stimulus-independent as well as transition probabilities which are stimulus-dependent. Both types have a different effect on the values of $p(x,t)$. The first type aligns all detectors on R^2_c and also the d states on R^2_d for every r in R^2_c. If the input is spatially uniform the space- and disparity-relationships of the probabilities are in the long run fully determined by the transition-probabilities R and S. If the input is not spatially uniform the input co-determines the disparity-perception. Then f weights its distant influence in the disparity space. Thus in principle it seems possible to estimate the range of f from distance influence of the input which is measured phenomenologically for fusion and disparity-depth. However, this estimation depends on the type of stimulus (Fender & Julesz, 1967) and on the criterion that subjects use (Duwaer, 1981). For instance, the estimation is much larger for line stereograms than for random-dot stereograms. The criterion which is important with respect to the presented theory, is the one for which the binocular interaction breaks down. The threshold for this interaction is

the highest threshold value which can be found in experiments on single and double vision. It seems reasonable (Duwaer & Van den Brink, 1981) to estimate these values between 4 and 10 min of arc.

One can assume that measurements with random dot stereograms are the most reliable measurements for a precise determination of the range of f. In contrast with other types of binocular stimuli the range for random-dot stereograms does not depend on the number of dots within the range but on the disparity itself. There is no remarkable difference in difficulty of fusion and depth perception if the resolution of a stereogram changes, even if the difference in resolution is a thousand times (Julesz, 1971, chapt. 5.4). Julesz (1971, pp. 175) therefore concluded

> "Since the random-dot stereoscopic images are devoid of monocular shapes, the binocular correlation between corresponding areas has first to be established. It seems that this labeling of corresponding points can occur only within Panum's visual region".

This region seems to be a circular disc with diameter of 6 min of arc. It is difficult to determine the shape of the function f, since only the disparity, for which a transition between fusion and nonfusion - whatever the criterion may be - occurs, can be measured. The fact that normal stereograms have a much larger region of fusion implies that the function f does not equal zero outside the circular disc, only that it must be smaller. Therefore it is assumed that f is a smooth function where Panum's area corresponds roughly to the spreading or the half-width.

An arbitrary choice for f has to be made. For the general theory here, its formula is not important. If one would like to formulate a model, one can choose for the time being a simple function, such as for instance a Gaussian distribution

$$f(d) = b \exp\left[\frac{-|d|^2}{2a^2}\right]$$

3.1.2. The range of the function T

Julesz (1971, ch. 5.9) showed that pulling the half-images of a stereogram apart with some speed causes transitory loss of perception of depth, because refusion occurs, the disparity difference of the two images lies within the fusional area. It holds for normal as well as for random-dot stereograms. The obvious experimental difference between fusion, which can be explained by a mechanism based on local rivalry and the phenomena of hysteresis and of dependency of pulling-speed is the basis for the splitting of the transition function S in Section 2.2.3.2 into a (input-dependent) rivalry-part and an input-independent part.

Apart from the so-called "fast"-pulling discussed above it is possible to pull the targets apart "slowly", so that transitory loss of perception does not occur. There is a definite loss of perception for some disparity value. Clearly the difference between this maximum input-disparity and the perceived disparity exceeds the fusional region. The value of this maximum depends on the stimulus. The existence of the boundary implies that $T(\mathbf{d},\mathbf{d}')$ is not a pure distance-function (of $|\mathbf{d}|$) but that it is really a function of $\mathbf{d}'$. Clearly the transition from $\mathbf{d}'$ to disparity values closer to the origin becomes easier than to those farther from it as $\mathbf{d}'$ withdraws from the origin. The fact, that the boundary of the loss of perception is much smaller (see Section 1.2.6.5) in the case of vertical displacement (20 min of arc) than in the case of horizontal displacement (1-2 deg), implies that the pulling to the origin as a function of d'_y increases much faster than the pulling as a function of d'_x.

Different disparity values at one place $\mathbf{r}$ in R^2_c can be perceived simultaneously, as shown by Julesz (1971, ch. 5.7). Thus if the range $|\mathbf{d}|$ of $T(\mathbf{d},\mathbf{d}')$ is be of the order of the perceivable disparities disparity-perception would not be sharp unless the strength of T is much weaker than that of f. However, the existence of the disparity to which the two half-images of a stereogram can be pulled apart implies that the strengths of T and f are of an almost equal magnitude. At the same time the disparity-perception is not influenced by this equal magnitude as

Julesz (1971, p. 174) reports:

> "At the point of break-away the loss of fusion was abrupt and total; at a lesser disparity by a minute amount the fusion was perfect with no hint of impending disaster".

Thus one is forced to the conclusion that the range of T is small with respect to the range of f. Probably its magnitude is in the range of the stereoscopic acuity, 16 sec of arc, at which point stereoscopic depth loses its clarity (Julesz & Spivack, 1967).

3.1.3. The range of the function R

Only a very small region of the fovea - 10-30 min of arc (Polyak, 1941; Julesz, 1971) - gives the sharpest vision. It decreases as the distance to the center of the fovea increases. However, there is no evidence that any phenomenon other than the structure of the retina contributes to these effects. Moreover, the perfect perception of different depth-planes in random-dot stereograms with a low percentage of black dots shows that the co-operative interaction is about equally strong and has an equal range outside the region of sharpest vision as inside (Julesz, 1971, ch. 4.5). These planes can take up an area of 16 x 16 square degree of arc and can be transparent even if a neutral plane is biased by the presence of monocular cues (Julesz, 1971, ch. 5.7).

Therefore, it is assumed that $R(r,r')$ is independent of r' in a stimulus-free space. In principle the assumption holds during stimulation of the eyes. However, it is easy to incorporate shifting, contraction and expansion of the percept in the theory by assuming that R is a function of r', the time t and the type of eye (left or right). These variables can depend on eye and head movements. It could explain why in free vision the perceived space remains stable under eye and head movements. Findings concerning aniseikonia can also be brought into the theory in this way. But, all this is not within the scope of this study, so for the moment only the assumption that R is a pure distance function

is useful. Because of the existence of a Vernier-acuity of 2 sec of arc. (Berry, 1948; Doesschate, 1955) the source of which is prior to or, at the level of, stereopsis (Julesz, 1971, ch. 3.6) the range of R has to be some sec of arc at most. It is supposed that R is a positive and continuous function of r, which decreases as |r| increases.

3.1.4. Summary

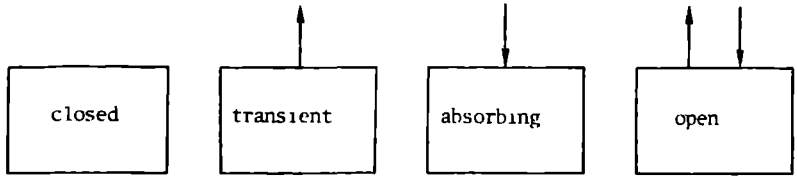
The range of the functions R, T, f and g are summarized in Table 3.1.

functions	T	R	f	g
range	16 sec	2 sec	6 min	0 sec
type of range	stereo-acuity	vernier acuity	panum's area	unkown

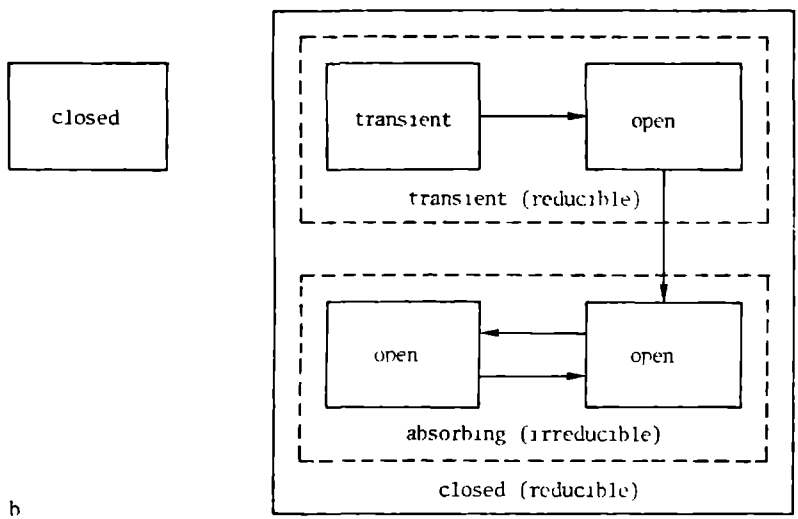
Table 3.1. The range of T, R, f and g.

3.2. Some types of solutions

Characteristics of the solutions for some special stimulus conditions can be derived from well known theorems in the theory of stochastic processes. Their formulations and proofs can be found in standard books on this topic, such as, for instance, Doob (1953). The most important theorems are summarized in Appendix C. These are sketched without discussion of the additional mathematical complications. It is supposed that the necessary mathematical requirements for the transition probability $q(x,t|x',t')$ and the probability-density $p(x,t)$ are fulfilled, among them that $p(x,t)$ is a continuous function of t for all x in X. The most important theorems concern the behaviour of a stochastic system if



a



b

Figure 3.1. Types of subsystems (a) and a reducible system (b).

time tends to infinity.

Sometimes the input to the rivalry system is such that it can be divided into subsystems, thereby allowing every subsystem to be studied independently of the other ones (see Figure 3.1). A subsystem can be such that some of its states can go over into other states of another subsystem, but that states of other subsystems cannot go over into its states. Then in the long run the probability, that one can find the

system in a state of the first subsystem, will become zero. Such a subsystem is called a transient subsystem. Such a probability of a subsystem of which the states cannot go over into states of other subsystems, but which can be reached by states of other subsystems will grow. It is called an absorbing subsystem. A subsystem, of which the states have no exchange at all with other subsystems, is called a closed subsystem. In the long run only the probabilities concerning closed and absorbing subsystems can differ from zero. Thus every solution of the systems equation tends to a linear combination of solutions of these subsystems.

A (sub)system, which cannot be divided into such closed or absorbing and transient subsystems is called irreducible. Every irreducible (sub)system has only one time-independent solution (see Appendix C), apart from a trivial factor. Such a solution is called a stationary solution. As far as the cyclopean system is concerned, it will be shown below for some special stimulus conditions that the percepts can be described as stationary solutions of the systems equation. The trick is that for each condition the probability density, P, and thus the systems equation (2.3.25), are reformulated so that the system is split into transient and absorbing subsystems. The quantitative characteristics of the solutions will be described in the next Chapters. It will be shown that these can be derived from the structural properties of the theory.

3.2.1. Open subsystems; Hysteresis

Unless the input of the system is zero, it does not have closed subsystems because of the range of the functions f, T and R. Then the question of whether an open subsystem is a transient or an absorbing one not only depends on the input but also on the actual state of the system (see Section C.2). In fact this property explains in general the appearance of hysteresis at psychophysical level. Whether it appears or not depends only on whether, for a given input, two disjunct sets of system states exist such that a subsystem changes from a transient one into an absonbing one if the actual state changes from one set into the other. Thus, although the dynamics of the system is a Markov process, henomena

can be observed which seem to be caused by non-Markovian processes, if subsystems change from transient to absorbing ones and vice versa.

3.2.2. Monocular stimulation

If the stimulus to one eye - for instance the left eye - equals zero and if the noise in the input of the cyclopean system can be neglected, $S_L(r,t)$ equals zero. Then it follows by substitution of $S_L = 0$ in the systems equation (2.3.25), that the subsystem which is generated by the bl- and dl-states is a transient one and the subsystem which is generated by the br- and dr-states is an absorbing one, if $S_R(r,t)$ does not equal zero for all r. So all P(bl,d,r,t) and P(dl,d,r,t) tend to zero if t tends to infinity. Thus for large t, equation (2.3.25) reduces to equation (3.2.1).

$$\frac{\partial}{\partial t}\begin{bmatrix}P(br,d,r,t)\\P(dr,d,r,t)\end{bmatrix} = \int dr'\{R(r-r',r')\begin{bmatrix}P(br,d,r',t)\\P(dr,d,r',t)\end{bmatrix} - R(r'-r,r)\begin{bmatrix}P(br,d,r,t)\\P(dr,d,r,t)\end{bmatrix}\} \tag{3.2.1}$$

$$+\int dd'\{T(d-d',d')\begin{bmatrix}P(br,d',r,t)\\P(dr,d',r,t)\end{bmatrix} - T(d'-d,d)\begin{bmatrix}P(br,d,r,t)\\P(dr,d,r,t)\end{bmatrix}\}$$

$$+\int dd' g(d'-d)\begin{bmatrix}S_R^+(r-d,t)P(dr,d',r,t)-S_R^-(r-d',t)P(br,d,r,t)\\S_R^-(r-d,t)P(br,d',r,t)-S_R^+(r-d',t)P(dr,d,r,t)\end{bmatrix}$$

Let

$$\begin{aligned} b(d,r,t) &= P(br,d,r+d,t)\\ D(d,r,t) &= P(dr,d,r+d,t)\end{aligned} \tag{3.2.2}$$

then the equation for B and D (3.2.3), which is in fact an equation for P(3,d,r+d,t), follows from (3.2.1)

$$\frac{\partial}{\partial t}\begin{bmatrix} B(d,r,t) \\ D(d,r,t) \end{bmatrix} = \int dr' \{ R(r-r',r'+d) \begin{bmatrix} B(d,r',t) \\ D(d,r',t) \end{bmatrix} - R(r'-r,r+d) \begin{bmatrix} B(d,r,t) \\ D(d,r,t) \end{bmatrix} \}$$

$$+ \int dd' \{ T(-d',d'+d) \begin{bmatrix} B(d+d',r,t) \\ D(d+d',r,t) \end{bmatrix} - T(d',d) \begin{bmatrix} B(d,r,t) \\ D(d,r,t) \end{bmatrix} \} \qquad (3.2.3)$$

$$+ \int dd' g(d') \begin{bmatrix} S_R^+(r,t) D(d+d',r-d',t) - S_R^-(r-d',t) B(d,r,t) \\ S_R^-(r,t) B(d+d',r-d',t) - S_R^+(r-d',t) D(d,r,t) \end{bmatrix}$$

Since $R(r,r')$ is independent of r', the co-operative interaction and the monocular rivalry are independent of d. Because of the second term at the right side, every time-dependent solution, of which d lies outside the range of T, tends to zero. Consequently the same holds for $P(br,d,r,t)$ and $P(dr,d,r,t)$. Thus for every solution of the original equation the disparity spread does not exceed the stereo-acuity. Or, in other words, in the case of monocular stimulation disparity depth is not perceived.

3.2.3. Identical stimulation

If the input functions from both eyes are equal, for instance because both stimuli are equal, $P(bl,-d,r,t)$ and $P(dl,-d,r,t)$ fulfil the same equation as $P(br,d,r,t)$ and $P(dr,d,r,t)$ respectively. This follows easily from the substitution of $S_L = S_R$ in the systems equation (2.3.25). The system is irreducible with respect to the R^2_d-space, so that it follows straightforwardly that $P(bl,-d,r,t)$ and $P(dl,-d,r,t)$ tend to $P(bl,d,r,t)$ and $P(dr,d,r,t)$ respectively. Consequently, the contribution of the binocular rivalry term after integration of the systems equation over d tends to zero. The functions

$$B(d,r,t) = P(br,d,r+d,t) + P(bl,-d,r+d,t)$$
$$D(d,r,t) = P(dr,d,r+d,t) + P(dl,-d,r+d,t) \qquad (3.2.4)$$

are described by an analogous equation as in the case of monocular stimulation (see Section 3.2.2) to which the term

$$\int dd'\, f(d'+2d)\begin{bmatrix} S_R^+(r,t)B(d+d',r-d',t) - S_R^+(r-d',t)B(d,r,t) \\ S_R^-(r,t)D(d+d',r-d',t) - S_R^-(r-d',t)D(d,r,t) \end{bmatrix} \qquad (3.2.5)$$

is added at the right-hand side. By carrying out summation (3.2.4) in the systems equation, it follows that the behaviour of the system is fully described by equation (3.2.1), combined with (3.2.5), if t tends to infinity. Again, the co-operative interaction and the monocular rivalry are independent of d. If B(d,r,t) and D(d,r,t) are zero almost everywhere if d lies outside the range of T then f(d'+2d) can be regarded as independent of d since the range of T is much smaller than the range of f (4.5 : 100). Thus the d-dependency is fully determined by the second term. It implies that B(d,r,t) and D(d,r,t) remain almost zero everywhere if d lies outside the range of T. Since the system is irreducible with respect to the R_d^2-space every solution will tend to such a solution. Thus in the case of identical input from (stimulation of) both eyes disparity depth will not be perceived.

Since disparity-depth is not perceived, P(j,d,r,t) can be replaced by its integral over d without changing the percept. The contribution of the rivalry term (and thus of the integral of (3.2.5)) to this function tends to zero (see above), so that the remaining equation equals the equation in the case of monocular stimulation.

3.2.4. Complementary stimuli

If the half-images are complementary, thus

$$S_R(r,t) = -S_L(r,t) \qquad (3.2.6)$$

arguments analogous to those in the Section above, lead to the conclusion that disparity depth will not be perceived. The equations are invariant for an exchange of the pairs

$$\begin{bmatrix} P(br,d,r,t) \\ P(dr,d,r,t) \end{bmatrix} \text{ and } \begin{bmatrix} P(dl,-d,r,t) \\ P(bl,-d,r,t) \end{bmatrix}$$

so that the equations for the sum and difference of these pairs are independent. Thus the rivalry between the eyes - the equation for the differences is independent of the "monocular" rivalry - the equation for the sum. Clearly the system is built up from at least two independent subsystems. The rivalry itself will be discussed in Chapter 5 and it will be argued that it cannot reach an unique stationary solution, due to fluctuations in the input.

3.2.5. Summary

The rivalry equation fulfils the basic requirement that the disparity for monocular, identical and complementary stimulation becomes zero if time tends to infinity. Identical and monocular stimulation with the same stimulus deliver a similar image, apart from a possible brightness-difference (see Chapter 4). Complementary stimuli evoke more than one stationary solution.

3.3. Spatio-temporal processes

Lowest order expansion of T and R leads to a second order differential equation. Then the co-operative interaction and the autonomous disparity interaction are described by the spatial part of the so-called Fokker-Planck equation (Van Kampen, 1960). Such a lowest order expansion

is meaningful, since the range of T and R is small (see Table 3.1) with respect to the mean stimulus size. Therefore the solutions of the expanded equation have to fit with data on spatio-temporal relations and the autonomous disparity-mechanism.

Since R(r,r') is independent of r' the lowest order expansion of R leads to a Wiener-process in the R_c^2 space, in any case in the absence of stimulation (see Section 3.1.3). One can calculate the effects of an image which is moved after it is stabilized on the retina. Indeed (see also Section 2.1.4.1), such a calculation *) shows the existence of darkness and brightness barriers, i.e. the field is in an antagonistic - brightness and darkness respectively - state.

The Wiener-process yields also that the strength of the influence of an input at r,t on the perception of an input at r',t' is a linear function of

$$\exp\left[\frac{-a(r-r')^2}{|t-t'|}\right]/|t-t'|$$

Such a linear function can be fitted with experimentally found U-shape curves in apparent motion (Buffart, 1978). These curves expresses the influence of one stimulus on another stimulus as a function of time.

Since in the absence of stimuli the autonomous disparity interaction drives d towards the origin (see Section 3.1.2), the lowest order expansion of T leads to an Ornstein-Uhlenbeck process in the R_d^2 space. The decay of d in this process can be fitted with data of Fender and Julesz (1967) on the fall-off of the disparity in the absence of stimulation (Buffart, 1978).

---

*) The calculation is laborious. In order to get an analytical result one can replace the Lorentz-distribution (0.1.10) by a Dirac-delta function. Then one can derive a recursive matrix equation, which can be solved.

4.1. Introduction

The term brightness is used for the phenomenon of light and dark perception as well as for the perception of light in contrast to the perception of dark. Since the context makes clear in which sense it is used this ambivalence does not require re-definition.

The literature on brightness and darkness shows a diversity of theoretical and experimental methods. The theories have one common feature; they are not related to any other direct perceptual phenomenon. In this Chapter it will be shown that the "rivalry" theory which has been developed in Chapter 2, serves as a basis for a special form of the centroid model for binocular brightness which has been described by De Weert and Levelt (1974). Since no special assumption has been introduced in the theory about the binocular brightness mechanism the centroid model is a pure prediction from the theory. It is based on a stationary solution of the model. De Weert and Levelt showed in their comparative study that the centroid model gives the best results. This is supportive for the theory here.

Curtis and Rule (1978) have thrown doubt upon the conclusion of De Weert and Levelt. They prefer a vector model for the binocular combination of monocular brightness. However, their criticism of the centroid model is unwarranted. De Weert and Levelt argued that monocular sensation value can be described by $(L+c)^n$ if L is the luminance and c the constant background luminance. The estimation of c ranges from 10^{-4} up to 10^{-2} cd/m^2 and that of n from 0.22 up to 0.44. Curtis and Rule calculated the relative decrease of monocular brightness with respect to binocular brightness on the basis of the centroid model if one or both eyes are stimulated with a luminance L. Therefore they used

$c = 10^{-4}$ cd/m^2 and n = 0.35. It appeared that the fall off ranged from 3.7% for L = 1 cd/m^2 up to 0.3% for L = 1000 cd/m^2. These values are far outside the range they have measured. This changes from 10% up to 24%. However, the choice of the values for c and n could not be worse. Other values of c and n within the ranges of De Weert and Levelt deliver per-

n	c/L	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}
.14	$(\frac{c}{c+L})^n$	0.52	0.38	0.28	0.20	0.15	0.11
	d (%)	16.4	17.1	15.6	13.3	10.8	8.5
.22	$(\frac{c}{c+L})^n$	0.36	0.22	0.13	0.079	0.048	0.029
	d (%)	16.9	14.1	10.3	6.7	4.4	2.7
.35	$(\frac{c}{c+L})^n$	0.20	0.089	0.040	0.016	0.008	0.0035
	d (%)	13.3	7.4	3.7	1.7	0.8	0.35
.44	$(\frac{c}{c+L})^n$	0.13	0.048	0.017	0.006	0.0023	0.0008
	d (%)	10.0	4.4	1.6	0.6	0.23	0.08
L ($\frac{cd}{m^2}$)	$c=10^{-2}$	1	10	100	1000		
	$c=10^{-3}$		1	10	100	1000	
	$c=10^{-4}$			1	10	100	1000

Table 4.1.1. Percentage of decrease (d) of monocular versus binocular brightness, following De Weert and Levelt's version (1974) of the centroid model.

centages of decrease which lie within the experimental range (see Table 4.1.1). Thus the argument for the rejection of the centroid model is not valid. Moreover the experimental results of Curtis and Rule are not reliable on the point of monocular brightness decrease. They did two series of brightness magnitude experiments, each with 15 subjects. The mean perceptual decrease in the first series was 24%. In the second one it was 10%. Thus they differ by a factor 2.4, which throws doubts on the validity of the experiments. A third objection to their arguments is that they also use a monocular sensation function L^n in their model. Their estimations of this n range from .07 up to .36 with a mean of 0.14. The relative monocular decrease for this n-value is also calculated in Table 4.1.1 Its values fall within the experimental range. Thus using their estimation for n, a good prediction for the monocular decrease follows from the centroid model. Another comment on the estimation of n is, that it is clearly not a subjective perceptual constant as the vector summation model supposes. The subjects of one brightness magnitude estimation experiment also took part in a category-rating experiment. A comparison between the two estimations of n for each subject shows that they sometimes differ up to a factor 2.5. This difference between the two experiments is not systematic; a set of 30 numbers between .07 and .23 seems to be randomly distributed over the 15 subjects and the two experiments. Obviously the value of n depends on influences that were not under experimental control. So the value of n serves only as a parameter to fit a descriptive formula with some data. However, the same can be done with other descriptive formulae, as Treisman (1970) showed for instance.

Clearly, it is possible to fit a formula as the best one for some set of experiments and another one as the best one for a set of other experiments. The formulae have mostly two or three parameters so that a good fit is always possible, unless a formula violates the known qualitative characteristics. Reviews can be found in Leveit (1965), Engel (1967, 1969), Blake and Fox (1973) and De Weert and Levelt (1974). All formulae are intended to describe a non-rivalrous stationary state solution of the visual system. However, it is impossible to set up an experiment in

which such a state is definitely reached. All experiments create a test-situation which is only an approximation to the situation that the formulae describe.

Every experiment on binocular brightness is a dynamic experiment or an experiment with rivalrous aspects. If a technique is used in which test and matching stimuli are presented successively, it is a dynamic experiment. If a technique is used in which test and matching stimuli are presented simultaneously, binocular rivalry is introduced because the two half-images cannot be form-identical. From the theory here it follows that in both cases monocular brightness is decreased. Successive as well as simultaneous presentation prohibits a full dominance of the partly monocular stimulus, although the full dominance in the case of monocular stimulation is supposed in every formula on binocular brightness. The departure from the full dominance depends on the experimental procedure. Thus the monocular brightness decrease can never be decisive for any model, unless one has some criterion to decide which part of the departure has to be ascribed to the brightness interaction itself.

Another problem in brightness measurement is caused by the fact that one does not know exactly what subjects are reporting. The subject's estimation of the brightness of an area can reflect its brightness or its lightness: a relation between its brightness and the brightness of other areas. As long as stimuli are used which consist of two luminance levels both reporting methods will give the same results. Thus only these types of experiments can be used as a test for a brightness model. The method of relative brightness judgement has been used by Jameson and Hurvich (1961). The matching field had an average luminance of 60 ml. The average luminance of the test field changed from .7 ml up to 8.9 ml. A subject is aware of the great difference in luminance if he changes from test to matching-field and vice versa. So he cannot report about the brightness itself. It is assumed that an estimation of the relative brightness is given. This quantity will be called lightness. The same term is used by Land (1964). However, brightness as well as lightness are defined here as perceptual quantities. Their physical counterparts are luminance and reflectancy. A model for lightness perception will be

introduced below. First a stationary state model for binocular brightness will be derived from the rivalry theory.

4.2. Brightness and darkness

In Section 2.1.6 it was argued that there are no equations for brightness and darkness spreading. This spreading coincides with spreading in the weighting system. A brightness or darkness impression arises because the units, having been activated into a certain state, adopt and transmit the strongest received signal belonging to this state. Thus for instance a unit in the state darkness-right transmits the strongest received darkness-right signal. A brightness (darkness) signal can be received from a neighbouring unit in a similar state or from the corresponding local input, depending which one is largest.

All this can be stated formally as follows. Let the subspace $V(j,d,r,t)$ of $R_c^2 xT$ be defined by the property: r',t' is an element of $V(j,d,r,t)$ if there is a continuous mapping V'' from $[0,1]$ onto $R_c^2 xT$ with

$$\begin{aligned} &V''(0) = r,t \\ &V''(1) = r',t' \\ &P(j,d,V''(v)) > 0 \text{ for all } v \text{ in } [0,1] \end{aligned} \qquad (4.2.1)$$

This last requirement yields that there is a pathway of units in the same state j,d from r',t' to r,t so that the unit at r,t can receive the input signal at r',t'. Thus V indicates the domain of units from which the unit at r,t can be reached by a signal corresponding to its state. The brightness $B(d,r,t)$ at r,t in $R_c^2 xT$ and for d in R_d^2 is defined by means of brightness and darkness functions of which the definitions below fulfil the requirements mentioned above. Let

$$
\begin{aligned}
B_{BR}(d,r,t) &= P(br,d,r,t)\ \max[S^{+}_{R}(r'+d,t')] \\
&\qquad\qquad r',t' \text{ in } V(br,d,r,t) \\
B_{DL}(d,r,t) &= P(dl,d,r,t)\ \max[S^{-}_{L}(r'-d,t')] \\
&\qquad\qquad r',t' \text{ in } V(dl,d,r,t) \qquad (4.2.2a) \\
B_{DR}(d,r,t) &= P(dr,d,r,t)\ \max[S^{-}_{R}(r'+d,t')] \\
&\qquad\qquad r',t' \text{ in } V(dr,d,r,t) \\
B_{BL}(d,r,t) &= P(bl,d,r,t)\ \max[S^{+}_{L}(r'-d,t')] \\
&\qquad\qquad r',t' \text{ in } V(bl,d,r,t)
\end{aligned}
$$

then (see also Table 2.2.3)

$$B(d,r,t) = B_{BR}(d,r,t) + B_{BL}(d,r,t) - B_{DR}(d,r,t) - B_{DL}(d,r,t) \qquad (4.2.2b)$$

and the total brightness B(r,t) at r,t equals

$$B(r,t) = \int dd\ B(d,r,t) \qquad (4.2.2c)$$

If B equals zero the brightness impression is "neutral". Darkness is experienced if $B < 0$, brightness if $B > 0$.

4.3. Noise; Eigengrau

From a perceptual point of view noise in the visual system is difficult to analyse. One type of noise is generated during the pathway from the source to the retina by physical noise, eye-movements, lens-distortion and accomodation. It will be left out of consideration since it is, for the most part, unimportant with respect to the topic of this thesis. The noise in the cyclopean system itself and, partly, the noise which is caused by eye-movements, is incorporated in the fact that the properties of an ensemble of cyclopean systems are calculated instead of those of one system. From electrophysiology it is known that spontaneous activity occurs in the visual system. It is a characteristic of complex

afferent systems. Jung (1959a,b) associates the spontaneous activity with the phenomenon of "Eigengrau" or "retinal gray", which is also called "Eigenlicht" (Wald, 1936). Spontaneous activity in the retina means, in terms of the theory presented here, that offset and onset signals occur spontaneously everywhere in the retinal system. It is assumed that the mean number of spontaneous offset signals in a retinal system always equals the mean number of spontaneous onset signals. Otherwise the "Eigengrau" cannot be a sufficiently stable phenomenon. It is assumed that the onset and offset signals are so distributed in time that it suffices to bring only their mean behaviour into account. The simplest assumption is that its contribution is a constant with strength S^o. That the spontaneous activity is influenced by the stimulus cannot be excluded but perceptually it is immeasurable. Thus S^o_L is added to S^+_L and S^o_R is added to S^+_R.

If stimulation is absent

$\begin{bmatrix} P(br,d,r,t) \\ P(bl,d,r,t) \end{bmatrix}$ fulfil the same equation as $\begin{bmatrix} P(dr,d,r,t) \\ P(dl,d,r,t) \end{bmatrix}$

Thus the stationary solution of both pairs is equal. It implies that in the stationary state it holds for the brightness

$$B_{BR}(d,r,t) = P(br,d,r,t)\, S^o_R = P(dr,d,r,t)\, S^o_R = B_{DR}(d,r,t) \qquad (4.3.1a)$$

Similarly it holds

$$B_{BL}(d,r,t) = B_{DL}(d,r,t) \qquad (4.3.1b)$$

Thus from 4.2.2b it follows

$$B(d,r,t) = 0 \qquad (4.3.1c)$$

when "Eigengrau" is observed, if the assumption about the relation

between spontaneous activity and "Eigengrau" is true. Furthermore it follows that dark-adaptation leads to the perception of "Eigengrau" in the absence of depth (see Section 3.2.3). Thus the state-values of a dark-adapted subject are such that disparity equals zero and the weighting-coefficients for both eyes and dark and bright are all equal.

4.4. Binocular brightness

In this paragraph it is shown that a form of the centroid model (see (4.4.4)) follows from the rivalry-theory as it is presented above. Let both half-images be form-identical such that

$$S_R(r,t) = sS_L(r,t) \qquad (4.4.1a)$$

where $s \geq 0$. The quantity $S(r,t)$, which is defined by (see also (0.1.2))

$$S(r,t) = \frac{S_R^2(r,t) + S_L^2(r,t)}{S_R(r,t) + S_L(r,t)} \qquad (4.4.1b)$$

expresses the form of the half-images, whereas s expresses their relative weigth. This can be seen in (4.4.3), which can be derived from (4.4.1) by using the definitions in (4.4.2).

$$s_R = \frac{2(1+s)}{1+s^2}$$
$$s_L = \frac{2s(1+s)}{1+s^2} \qquad (4.4.2)$$

$$S_R(r,t) = \frac{s_R S(r,t)}{2}$$
$$S_L(r,t) = \frac{s_L S(r,t)}{2} \qquad (4.4.3)$$

$$\frac{S_R^2+S_L^2}{S_R+S_L} = \frac{4(1+s)^2(1+s^2)}{(1+s^2)^2} * \frac{1+s^2}{2(1+s)^2} = 2 \qquad (4.4.4)$$

The relation (4.4.4) between S_R and S_L follows from (4.4.2).In Appendix D the brightness, which is evoked by the retinal output (4.4.1), is proved to be almost constant. This proof is based on some perturbation-approach for small values of m or |n| (see (4.4.5)).

$$n = \frac{1-s^2}{1+s^2}$$
$$m = \frac{2s}{1+s^2} \qquad (4.4.5)$$

Small values of n mean that $s \equiv 1$, thus $S_R(r,t) \equiv S_L(r,t)$. Small values of m mean that s or 1/s is small. It allows the intensity of one of the half-images to be small compared with the other one. It is an approach to monocular stimulation. It is proved, that for small m

$$B(r,t) = 2K(r,t) \qquad (4.4.6a)$$

and for small n

$$B(r,t) = 2\bar{K}(r,t) + \frac{n^2}{1+m}[\bar{L}(r,t)-\bar{K}(r,t)] \qquad (4.4.6b)$$

where $K(r,t)$, $\bar{K}(r,t)$ and $\bar{L}(r,t)$ are functions of $S(r,t)$, but not of s (see (D.18)). Thus a very interesting result has been obtained. The perceived brightness is constant, if $S(r,t)$ is constant, up to, and including, the first order of the parameters, - m and n -, of the perturbation approaches. This constraint on $S(r,t)$ converts (4.4.1b) into the centroid model, which is also expressed by (4.4.4). In other words the theory predicts binocular brightness constancy if the output strengths of both eyes predict constancy following the centroid model. It is not proven that (4.4.1b) is the formula for binocular brightness mixing. Generally (4.2.2) holds.

Levelt (1968) conducted experiments on binocular brightness constancy with form-identical stimuli. An experimenter has set the luminance of one of two half-images on a certain value. An observer has to adjust the luminance of the other one such that the binocular brightness equals the binocular brightness of two half-images with an equal and fixed luminance level. The alternation of both pairs of half-images was regulated by the observer. Every half-image consisted of only two luminance levels.

The quantities S_R, S_L and S (see (4.4.1) and (4.4.3)) are determined by the retinal system and the stimuli. If in each half-image two luminance levels - L_{R1}, L_{R2}, L_{L1}, L_{L2} - are present, the quantities S_R and S_L are following Section 0.1.3 proportional with respectively

$$S_R \sim \frac{L_{R1}}{L_{RO}+L_{R1}} - \frac{L_{R2}}{L_{RO}+L_{R2}}$$
$$S_L \sim \frac{L_{L1}}{L_{LO}+L_{L1}} - \frac{L_{L2}}{L_{LO}+L_{L2}} \qquad (4.4.7a)$$

If the lowest levels can be assumed to equal zero, these quantities can be written respectively as

$$S_R \sim \frac{L_R}{L_{RO}+L_R}$$
$$S_L \sim \frac{L_L}{L_{LO}+L_L} \qquad (4.4.7b)$$

Assuming that the adaptation in the retinal system is slow in comparison with the alternation rate in Levelt's experiments, the L_{LO} and L_{RO} remain constant during the matching procedure and will have some mean value with respect to the two stimulus pairs (see Section 0.1.3). Substitution in the equal luminance function (4.4.1) delivers

$$\frac{\left(\frac{L}{L_{RO}+L_R}\right)^2+\left(\frac{L}{L_{LO}+L_L}\right)^2}{\left(\frac{L}{L_{RO}+L_R}\right)+\left(\frac{L}{L_{LO}+L_L}\right)} = \frac{\left(\frac{L}{L_{RO}+L}\right)^2+\left(\frac{L}{L_{LO}+L}\right)^2}{\left(\frac{L}{L_{RO}+L}\right)+\left(\frac{L}{L_{LO}+L}\right)} \qquad (4.4.8)$$

where L is the luminance of the two reference half-images.

If the adaptation parameters are fully determined by this luminance they would equal $e_R L^{.6}$ and $e_L L^{.6}$ respectively (see Section 0.1.3). The e_R and e_L are constants. If they are chosen to equal the parameter - $e_R = e_L = 2.14$ - that is derived from the experiments of König and Brodhun (1889) (see Section 0.1.1) the equibrightness equation (4.4.8) becomes

$$\frac{\left(\frac{L}{L_{RO}+L_R}\right)^2+\left(\frac{L}{L_{LO}+L_L}\right)^2}{\left(\frac{L}{L_{RO}+L_R}\right)+\left(\frac{L}{L_{LO}+L_L}\right)} = \frac{L}{L_0+L} \qquad (4.4.9)$$

with $L_{RO} = L_{LO} = L_0 = 2.14\, L^{.6}$. In Figure 4.4.1 normalized equibrightness curves - viz. $L_L:L$ as a function of $L_R:L$ - have been plotted for three values of L. They are precisely the type of curves that Levelt (1968) found experimentally.

However, adaptation not only occurs to the luminance of the reference images, but also to the luminance of the test images. Consequently, L_R will be greater than L if L_L equals zero. This effect is also found by Levelt. It is used as an argument in favour of vector summation models for binocular brightness mixing (Curtis and Rule, 1978; Engel, 1967). But according to the model presented here it is caused by improper retinal adaptation, due to the experimental technique. The process of this retinal adaptation is not sufficiently known to draw conclusions about the actual adaptation values. For the moment it can only be concluded that the actual adaptation is determined by the adaptation to the reference image as well as to the test image. Two simple formulations of this phenomenon (see Section 0.1.3) are

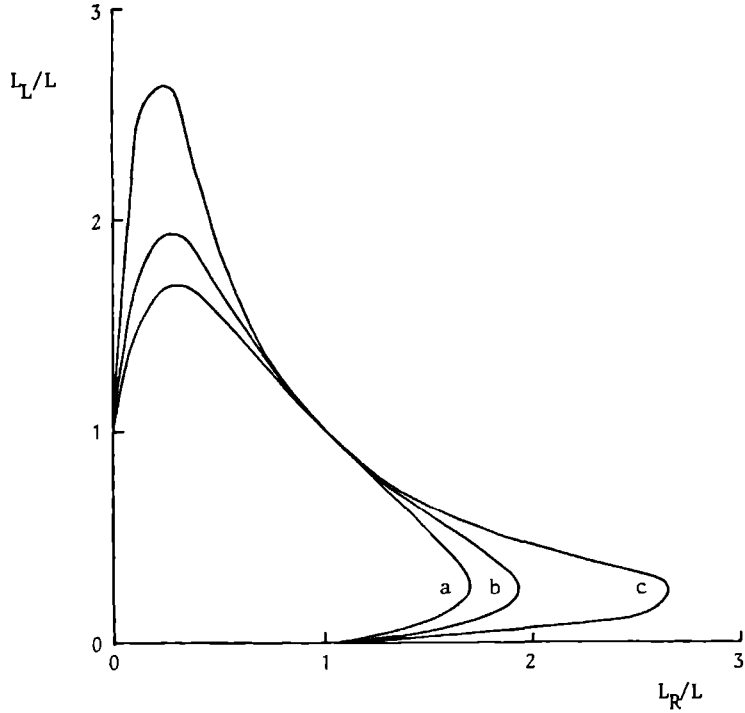


Figure 4.4.1. Normalized binocular equibrightness curves with constant adaptation: a. L=15 cd/m^2; b. L=30 cd/m^2; c. L=75 cd/m^2.

$$L_{LO} = e_L(aL_L^{.6} + bL_L^{.6}) \qquad (4.4.10a)$$

$$L_{RO} = e_h(aL_R^{.6} + bL_R^{.6})$$

and

$$L_{LO} = e_L(aL + bL_L)^{.6}$$
$$L_{RO} = e_R(aL + bL_R)^{.6} \qquad (4.4.10b)$$

The coefficients can be chosen such that substitution of L_{LO} and L_{RO} in (4.4.8) delivers a nice theoretical representation of the experimental results *). This simulation, as such, does not have any value for the theory, because if one can look for two well chosen variables, a good fit can almost always be found. However, the fact that the centroid model can be derived from a general theory of local rivalry in binocular interaction is important. It is equally important that the experimental deviation from the model, as it is represented by formula (4.4.9), for low luminance values in one of both eyes, can be explained from retinal adaptation. Moreover, substitution of each of the three widely known types of transducer functions (see Section 0.1.1) produces (De Weert & Levelt, 1974) the type of equibrightness curves which have been found by Levelt (1968). In Figure 4.4.2 normalized equibrightness curves have

*) In Figures 4.4.3 and 4.4.4 Formula (4.4.10a) is used with a=1.5 and b=.7 for subjects jb. and wl., and, with a=2. and b=1.8 for subject hv. The meaning of these numbers is not clear. The extreme values in the curves are very sensitive for small changes in the adaptation parameters. It is remarkable that these values can also fluctuate stongly during experiments (De Weert, private communication). It seems reasonable to suppose that a and b depend on the strategy of a subject. The fact that a+b≠1, suggests that e_L and e_R are subject dependent. Subject hv. shows a non-consistent behaviour in the constants, if one looks for the best fitting curve for every experiment apart. It can be due to an inequality of e_L and e_R. Here a and b are chosen to equal the mean of the best values for each experiment. Subject jb. shows clear deviating constants for L = 30 cd/m^2 in Figure 4.4.3. The data in this experiment show a remarkable larger variability than the data in the other tasks. It can indicate a change of strategy. Since the cause is an open question, the a and b values are chosen to equal the values of the other three cases.

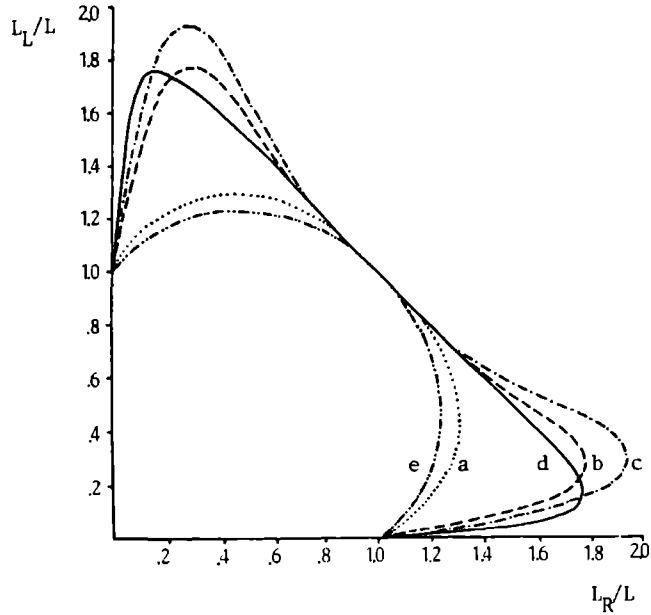
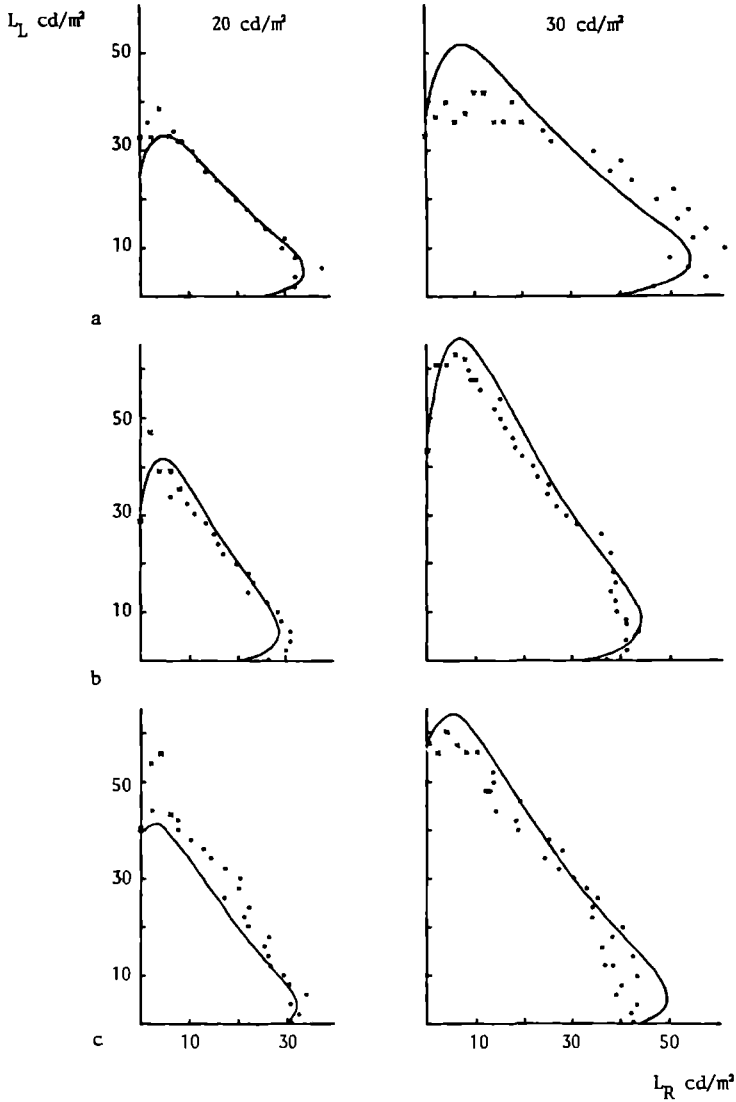


Figure 4.4.2. Normalized theoretical equibrightness curves based on different transducer functions: a. logarithmic function; b and c. quasi-logarithmic function with matching luminance 20 and 30 cd/m^2 respectively; d and e. power function with n = .33 and .90 respectively.

See opposite page:

Figure 4.4.3. Experimental (Levelt, 1968) and theoretical equibrightness with L = 20 cd/m^2 and L = 30 cd/m^2 respectively for three subjects: a. subject jb, m=.99; b. subject wl, m=1.13; c. subject hv, m=1.08. See also footnote *).


L_L cd/m²
20 cd/m²
30 cd/m²
50
30
10
a
b
c
10
30
50
L_R cd/m²

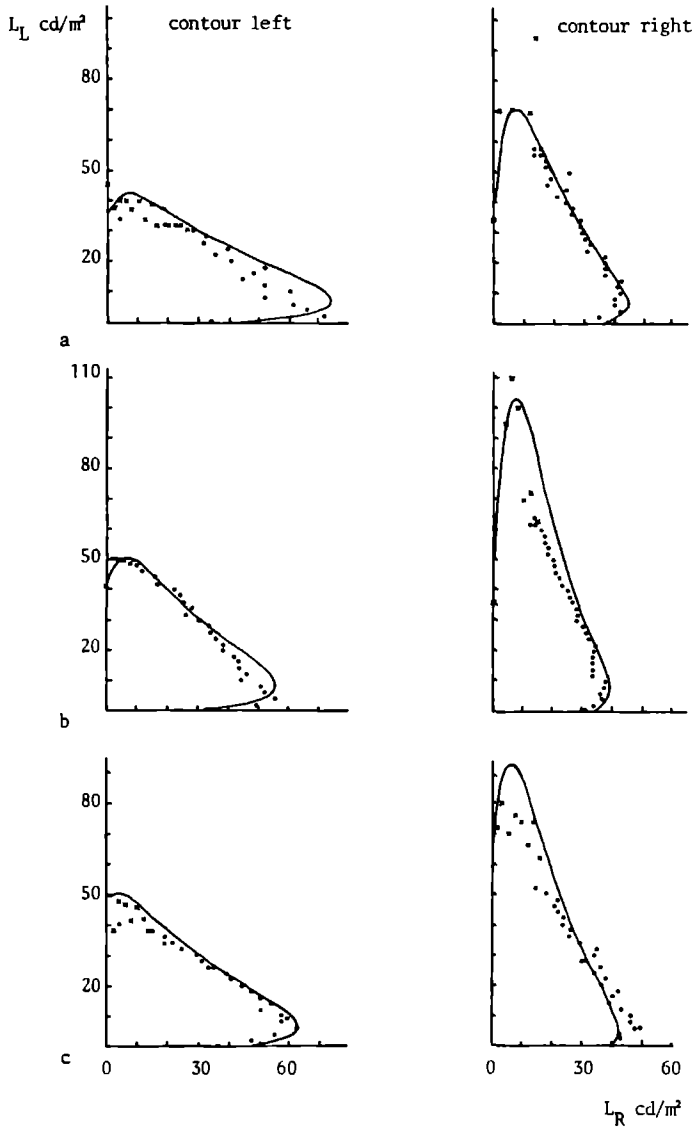
been plotted for the three types of transducer functions together. In principle, the predictions of the centroid model seem to be independent of the type of transducer functions. Clearly they are indistinguishable at least if the results for low luminance values for one of both eyes is left out of consideration (see also Section 0.1.1). However, it appears that only the quasi-logarithmic function (0.1.5) can cope with the results for these values in a satisfactory way (see also Figure 4.4.3).

The binocular equibrightness curves found in experiments are mostly non-symmetrical (Levelt, 1968). The obvious explanation is the existence of eye dominance. What this means in terms of the model is not clear. There are two ways of incorporating eye dominance in the model. Firstly, one can suppose that there is a differing dependency of both retinal systems on the luminance, which means that e_R and e_L differ. Simulation shows that this assumption does not lead to the desired results. Secondly, one can suppose that the cyclopean input from both eyes is not of equal strength. It would mean that the input from the right eye is a factor m stronger than the input from the left eye if both half-images are identical. This means that s_R in formula (4.4.2) is replaced by ms_R. Substitution in the equibrightness formula (4.4.8) produces

$$\frac{m^2\left(\frac{L_R}{L_{RO}+L_R}\right)^2+\left(\frac{L_L}{L_{LO}+L_L}\right)^2}{m\left(\frac{L_R}{L_{RO}+L_R}\right)+\left(\frac{L_L}{L_{LO}+L_L}\right)}=\frac{m^2\left(\frac{L}{L_{RO}+L}\right)^2+\left(\frac{L}{L_{LO}+L}\right)^2}{m\left(\frac{L}{L_{RO}+L}\right)+\left(\frac{L}{L_{LO}+L}\right)} \qquad (4.4.11)$$

See opposite page:

Figure 4.4.4. Experimental (Levelt, 1968) and theoretical equibrightness curves with $L = 30$ cd/m^2 and extra contour information in right ($n=1.7$) and left eye ($1/n$) respectively for three subjects: a. subject jb; b. subject wl; c. subject hv. See for the other constants the caption of Figure 4.4.3.


L_L cd/m²
contour left
contour right
80
50
20
a
110
80
50
20
b
80
50
20
c
0
30
60
0
30
60
L_R cd/m²

For every observer the parameter m can be found by looking for the best fitting theoretical curve. Experimental and theoretical results for three observers of Levelt (1968) are represented in Figure 4.4.3.

Levelt (1968) also did measurements on binocular equibrightness with an extra contour - with the same physical contrast - in one of both eyes. Following the theory above, this only multiplies the weighting coefficients by some coefficient n. If extra monocular contour information is presented to the right eye its weighting coefficient has to be multiplied with n. This can be proved in a similar way as in Appendix D by multiplying the right-hand side of $P(br,d,r,t)$ and $P(dr,d,r,t)$ in (D.2) with n. Then it follows that brightness constancy occurs if s_h^2 is replaced by ns_h^2 and s_h by ns_R. Thus formula (4.4.11) becomes

$$\frac{nm^2\left(\frac{L_R}{L_{RO}+L_R}\right)^2+\left(\frac{L_L}{L_{LO}+L_L}\right)^2}{nm\left(\frac{L_R}{L_{RO}+L_R}\right)+\left(\frac{L_L}{L_{LO}+L_L}\right)} = \frac{nm^2\left(\frac{L_R}{L_{RO}+L_R}\right)^2+\left(\frac{L_L}{L_{LO}+L_L}\right)^2}{nm\left(\frac{L_R}{L_{RO}+L_R}\right)+\left(\frac{L_L}{L_{LO}+L_L}\right)} \qquad (4.4.12)$$

This makes it possible to test the model, because, if n is fitted for half-images with an extra contour in the right eye, the formula in which n is replaced by 1/n delivers the predictions for the case in which the left half-image contains the extra contour. The experimental and theoretical curves are given in Figure 4.4.4.

4.5. Lightness

As it has been argued in Section 4.1 it is generally unclear whether or not a subject follows the instructions if he has to compare the brightness or lightness of a test and matching stimulus. This problem does not exist if only two luminance levels are present in each of two form-identical half-images or if a brightness comparison is prevented, as in the experiments of Jameson and Hurvich (1961).

Monocular brightness experiments have a long history which started

with Hess and Pretori (1894). The experimental methods have changed considerably since then. At first test and matching stimuli were presented simultaneously to the same eye. This evokes interactions between test and matching stimuli so that the results are difficult to interprete. Fry and Alpern (1953) proposed a dichoptic matching method. The test and inducing fields are presented to one eye, the matching field to the other one. They suggested that interactions between the matching and the other stimuli do not occur with such a method. This reasoning can be accepted as far as the interactions are contrast effects as shown by Asher (1950). However, the existence of intraocular effects has been demonstrated by Fiorentini and Radici (1956) and Levelt (1968). Two types of binocular interaction can exist. The first type can be described as a permanent rivalry between the eyes. It means that full dominance can never be ascribed to one of both eyes even if the patterns from both eyes are spatially separated. The measure of dominance depends on the size of this separation and on the amount of contrast in both eyes. Because the dominance varies together with the brightness it is impossible to distinguish between both effects.

The second interaction type exists if a subject reports about lightness instead of brightness. The surroundings of test and matching fields overlap and are a combination of the contrast from both eyes. This is demonstrated by the fact that the half-images can be fused by means of a fixation point in both surroundings. Thus lightness is a function of the contrast in both eyes. This intermingling of effects makes it impossible to use the experimental results as a test for the theory presented here, because they cannot falsify it. One would have to calculate all spatial and temporal relations, a task which would require a special study. Furthermore one would have to take into account effects that are evoked in the retinal system. Thus Heinemann (1955) found brightness enhancement that looked like the enhancement found, for instance, by Bartley (1938, 1951, 1961) as a result of flicker and intermittent stimulation. In the cat's retinal system Grüsser and Creutzfeld (1957) found similar electro-physiological effects which suggest a retinal origin of enhance-

ment.

Experiments in which, by avoiding an interningling of lightness and brightness effects, one can expect that spatial and temporal relations do not play a significant role offer a better possibility of falsification. Such experiments are monocular experiments - or experiments with identical stimulation of both eyes - in which test and matching stimuli are not presented simultaneously. Furthermore these two have to differ considerably in brightness, and effects of order and duration of presentation must be avoided. Because the experiments of Jameson and Hurvich (1961) alone fulfil these requirements, we will use them as a test for the theory. All theoretical parameters will be chosen from the binocular experiments described in the Section above. Jameson and Hurvich (1961) asked for brightness judgments on the squares in the stimulus, represented in Figure 4.5.1. The stimulus is presented in three different light intensities, such that the ratios of the different illuminations of the six parts of the stimulus are always the same. The ratio between the background and the squares is unknown. That of the squares with respect to each other is $L_1:L_2:L_3:L_4:L_5 = 27:21.5:10.8:2.5:1$.

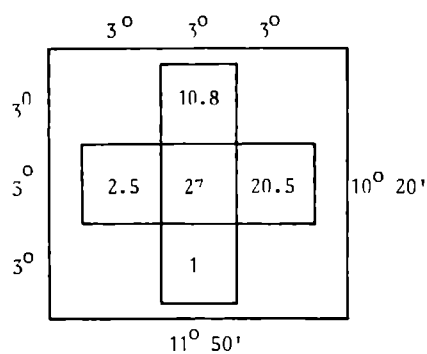


Figure 4.5.1. Test stimulus used by Jameson and Hurvich (1961).

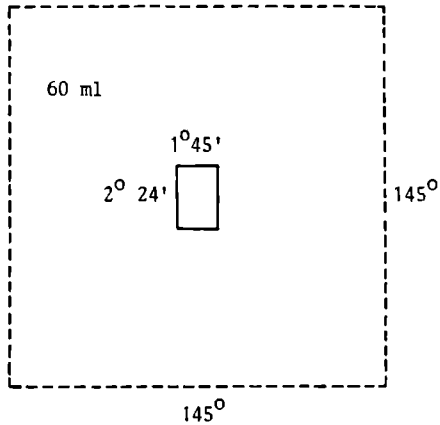


Figure 4.5.2. Matching stimulus used by Jameson and Hurvich (1961).

The most important results of the experiments are as follows: that brightness in such a complicated stimulus neither varies according to a power law, nor is it a function of the physical contrast within a stimulus; that the 5th square decreases in brightness if the illumination increases; that the brightness of the 4th square remains constant - possibly decreasing slightly and subsequently rising - and that the brightness of the remaining three squares increases with increasing illumination.

The matching stimulus - see Figure 4.5.2 - differs from the test stimulus in form, spatial proportions and average illumination. Moreover, the subject has to carry out a spatially complicated action in order to switch from the test stimulus to the matching stimulus and vice versa; he has to turn around. The theory does not deal with such complicated transformations. Indeed, quantitatively, the results are at first sight difficult to interpret, but in the supposition that the reported brightness is a monotonous function of the perceived brightness, they may be satisfactorily interpreted qualitatively. If one supposes that the diffusion-term in the systems equation evokes, in each square and in the background, an almost homogeneous spread of the effects evoked by

the contrast between both squares, one can approximate these effects as follows. Let the luminance of two adjacent areas equal L_1 and L_2. Let $L_1 > L_2$. Let S_{12} represent the output of the retinal system evoked by the luminance difference of both regions if the subject moves his eyes from area 2 to area 1. Then from Section 0.1.3 it follows that

$$S_{12} \sim C_{12} \qquad (4.5.1)$$

$$C_{12} = \frac{L_1}{L_0+L_1} - \frac{L_2}{L_0+L_1}$$

Let $\underline{b}_i$ and $\underline{d}_i$ (i = 1,2) represent the brightness and darkness weighting coefficients of both areas (1 and 2) before such a movement and let b_i and d_i (i = 1,2) represent them after a movement. Then it follows from the monocular rivalry term - (3.2.5) disappears after integration over d - in the systems equation for identical stimulation (3.2.1) that

$$b_1 - \underline{b}_2 = wC_{12}\,\underline{d}_2 \qquad (4.5.2)$$

$$d_2 - \underline{d}_1 = wC_{12}\,\underline{b}_1$$

The coefficient w is determined by the function g of the monocular term and the function S in (0.1.2). Assuming that all these functions are normalized functions, w can be set equal to 1 because of the homogeneity. (4.5.4) follows from (4.5.2) and (4.5.3), which holds by definition. (4.5.5) follows from (4.5.4).

$$b_i + d_i = \underline{b}_i + \underline{d}_i = 1 \qquad (4.5.3)$$

$$b_1 = C_{12} + (1-C_{12})\underline{b}_2 \qquad (4.5.4)$$

$$b_2 = (1-C_{12})\underline{b}_1$$

$$b_1 - b_2 = C_{12} - (1-C_{12})(\underline{b}_1 - \underline{b}_2) \qquad (4.5.5)$$

If it is assumed that in a situation where free scanning is permitted

the brightness levels and thus the differences $b_1 - b_2$ are held constant approximately, thus

$$b_1 - b_2 = \underline{b}_1 - \underline{b}_2 \tag{4.5.6}$$

then (4.5.7a) follows from (4.5.5). By replacing C_{12} in the denominator of (4.5.7a) by $|C_{12}|$ the formula (see (4.5.7b)) also holds if $L_1 < L_2$ (see (4.5.1)).

$$b_1 = b_2 + \frac{C_{12}}{2 - C_{12}} \tag{4.5.7a}$$

$$b_1 = b_2 + \frac{C_{12}}{2 - |C_{12}|} \tag{4.5.7b}$$

If area 2 has more - say n - adjacent areas with luminances L_i ($i = 1,n$) the weight of the transition from region 2 to region 1 is proportional to the mean number of the concerning eye-movements. It is reasonable that the latter are proportional to the relative length of the common border between the two areas in relation to the total length of the border of area 1.

$$v_{1i} = \frac{l_{1i}}{\sum_{j=1}^{n} l_{1j}} \tag{4.5.8}$$

If l_{1i} is the length of the border between area 1 and area i and if v_{1i} is its relative length (see (4.5.8)) with respect to the total length then, analogously to (4.5.7b), it holds

$$b_1 = \sum_{j=2}^{n} v_{1j} \left(b_j + \frac{C_{1j}}{2 - |C_{1j}|}\right) \tag{4.5.9}$$

Because none of the coefficients are zero, the maximum input for brightness transmission (see Section 4.2) is everywhere the same (4.2.2). This holds similarly for the darkness transmisson. Furthermore it follows from Section 0.1 that both maxima are equal - say M. Thus

(see (4.2.2) and (4.5.3)) the perceived brightness B_i of area i equals

$$B_i = (b_i - d_i)M = (2b_i - 1)M \qquad (4.5.10)$$

Since (4.5.10) holds for all areas the relative brightness is independent of M.

The main results of Jameson and Hurvich (1961), increasing and decreasing brightness as a function of lumination, can be predicted from the theory without any assumption about relative brightness. Two theoretical results are shown in Figure 4.5.3.

However, in order to make any quantitative prediction from the theory the notion of relative brightness or lightness has to be introduced. The theoretical lightness of an area is defined as the ratio of its brightness and the weighted sum of the brightness of neighbouring areas. The weighting coefficients are again the relative border lengths. The

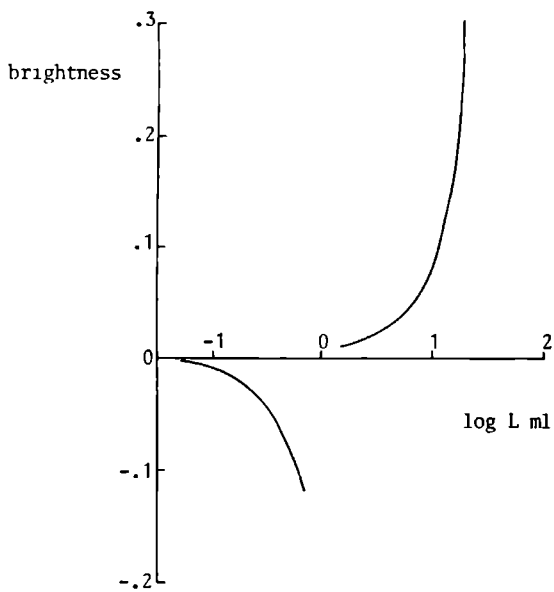


Figure 4.5.3. Calculated brightness as a function of lumination.

lightness L^1 of area 1 is given in (4.5.11) using (4.5.9) and (4.5.10).

$$L^1 = \frac{B_1}{\sum_{j=2}^{n} v_{1j} B_j} = \frac{2b_1 - 1}{\sum_{j=2}^{n} v_{1j} (2b_j - 1)} \qquad (4.5.11)$$

Such lightness values are calculated for all areas in the experiment of Jameson and Hurvich, and for the matching stimulus. From these a theoretical prediction could be made about the luminances that a subject would give to the matching stimulus assuming that he chooses the luminance of the matching stimulus such that its lightness equals that of the test stimulus. The results are given in Figure 4.5.4. In Figure 4.5.5 the theoretical brightness and the theoretical lightness of the matching stimulus are given as a function of its luminance. They show a

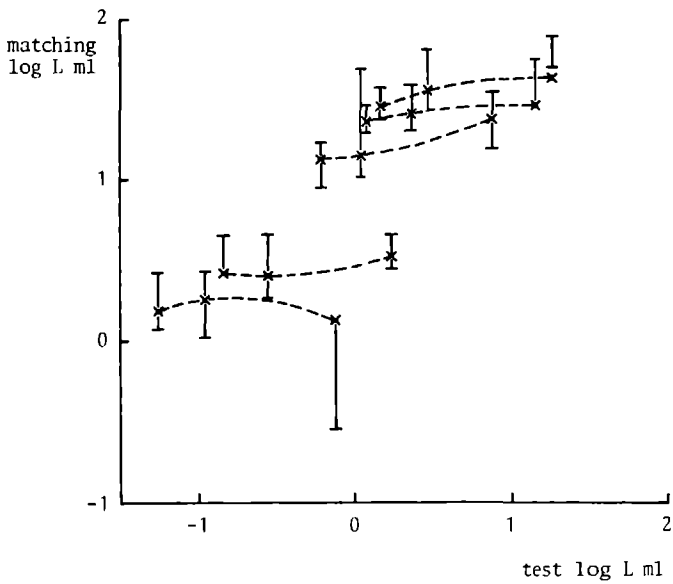


Figure 4.5.4. Comparison of experimental (Jameson and Hurvich (1961)) data (I) and theoretical predictions (-------).

saturation effect which has been reported by Jameson and Hurvich (1961).

Clearly the notion of lightness as it is introduced here, suffices to give good predictions for the results of a seemingly intricate experiment. Apart from this extra assumption the results are fully based on the rivalry theory without being fitted to any parameter.

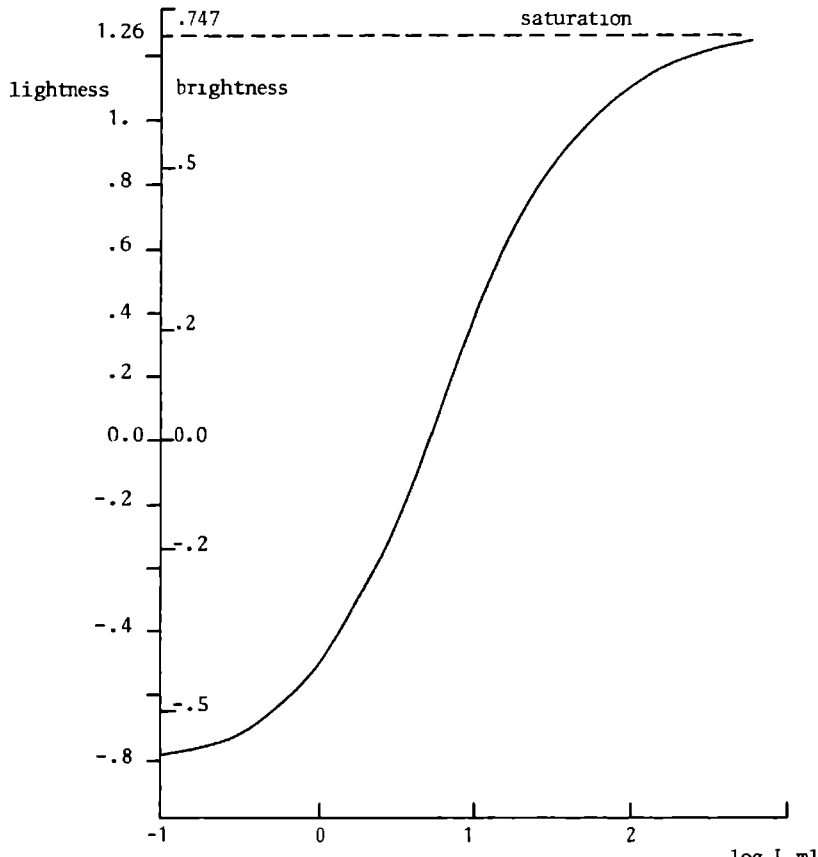


Figure 4.5.5. Calculated brightness and lightness of the matching stimulus.

5.1. Levelt's model for alternation

Levelt (1968) proposed a model for the alternation in global binocular rivalry. This model explains the perceptual dominance of one of the half-images and the alternation frequency of the dominance change on the basis of one notion: stimulus strength. This unification of both effects has not been proposed before. Let s_R and s_L represent the stimulus-strength in the right and the left eye respectively. Let $\bar{t}_R$ and $\bar{t}_L$ be the mean time during which respectively the right image and the left image is uninterruptedly perceived.

$$\bar{T} = \bar{t}_R + \bar{t}_L \qquad (5.1.1)$$

represents the inverse of the alternation frequency. The model states that $\bar{t}_R$ is a monotonously decreasing function h of s_L,

$$\bar{t}_R = h(s_L) \qquad (5.1.2a)$$

and likewise

$$\bar{t}_L = h(s_R) \qquad (5.1.2b)$$

Thus the alternation frequency

$$\frac{1}{\bar{T}} = \frac{1}{h(s_L)+h(s_R)} \qquad (5.1.3)$$

and the relative dominance of the right and the left image is given by respectively

$$D_h = \frac{h(s_L)}{h(s_L)+h(s_h)}$$

and (5.1.4)

$$D_L = \frac{h(s_R)}{h(s_L)+h(s_R)}$$

Thus an increase in one or both stimulus strengths increases the alternation-frequency. An increase of the stimulus strength of the right (left) eye decreases the mean dominance time of the left (right) image without changing the mean dominance time of the right (left) image. This cross-wise effect has been proved experimentally (Levelt, 1968). However, experiments can be interpreted only in terms of this model if the notion (change of) stimulus strength has been related to some (manipulation of) stimulus properties. Levelt introduced two operational anchor points for it. Firstly, the stimulus strength in a point is a decreasing function of the angular distance from the contours in a test pattern. Levelt reviewed the literature on this point and showed that the model is able to explain the experimental results on image dominance and alternation in which this anchorage has been used. Furthermore he executed two experiments on this point and showed that the results were in agreement with the predictions. He did the same for the second anchorage. This operational anchorage is the contour strength which has been defined as the ratio of physical contrast to the difference threshold. It is a practical definition because all effects due to retinal inhomogeneity, non-linearity and contour sharpness are comprised in the difference threshold whereas the contrast-term accounts for the luminance differences at the contour. From a theoretical point of view the difference threshold corrects for a lack of knowledge about the transformations from stimulus luminance distribution up to the cyclopean system.

It will be argued in the next Sections that the output of the retinal system $S(r,t)$ as introduced in the introduction is the equal of Levelt's stimulus strength s. It will be argued that $S(r,t)$ has the same

properties as s. Furthermore the question of how complementary stimuli can give rise to global rivalry will be discussed. It will be shown that its characteristics are influenced by several unknown factors, for instance eye-movements and blinks. Moreover, the systems equation (2.3.25) is unsolvable and a satisfactory approximation of its solution for the case of complementary stimuli could not be found. Therefore an exact calculation of, for instance, the alternation-frequency cannot be given.

5.2. The origin of global rivalry

In Section 3.2.4 it has been argued that in the case of pure complementary stimuli the cyclopean system is constructed from at least two independent subsystems. One governs the quantity

$$\begin{aligned} B(+,d,r,t) &= P(br,d,r,t) + P(dl,-d,r,t) \\ D(+,d,r,t) &= P(dr,d,r,t) + P(bl,-d,r,t) \end{aligned} \qquad (5.2.1)$$

and the other the quantity

$$\begin{aligned} B(-,d,r,t) &= P(br,d,r,t) - P(dl,-d,r,t) \\ D(-,d,r,t) &= P(dr,d,r,t) - P(bl,-d,r,t) \end{aligned} \qquad (5.2.2)$$

The first quantity is governed by R, T and a "monocular interaction" which equals

$$\begin{bmatrix} B(d) \\ D(d) \end{bmatrix} = \int dd' \{f(d+d')+g(d-d')\} \begin{bmatrix} S_R^+(r-d,t)D(+,d',r,t) - S_h^-(r-d',t)B(+,d,r,t) \\ S_h^-(r-d,t)B(+,d',r,t) - S_R^+(r-d',t)D(+,d,r,t) \end{bmatrix} \qquad (5.2.3)$$

As it has been argued in Sections 3.2.3 and 3.2.4 disparity-depth will not be perceived in the case of complementary stimuli. The d-dependency of the solutions is determined by T, which causes a tendency

to the origin (see Section 5.2.2). The range of T is probably 16 sec of arc so that the d variability of the solutions is smaller than 16 sec of arc. Therefore in the stationary case the sum f(d+d')+g(d-d') is a single peak function with a range only somewhat broader than g(d-d'). Thus in the stationary case B(+,d,r,t) and D(+,d,r,t) are comparable with P(br,d,r,t) and P(dr,d,r,t) in the case of monocular stimulation of the right eye. A stationary solution of the second subsystem is $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ which occurs if

$$\begin{aligned} P(br,d,r,t) &= P(dl,-d,r,t) \\ P(dr,d,r,t) &= P(bl,-d,r,t) \end{aligned} \qquad (5.2.4)$$

If it was a unique stationary solution one would expect that a homogeneous gray field is perceived. However, in contrast with the case of identical stimulation - with or without disparity - it is not a unique "stationary" solution if fluctuations are present in the retinal outputs. Generally, fluctuations are caused by eye-movements and blinks. Moreover, in experimental situations stimuli are never fully complementary because they contain, at least, some fusion contour.

As it will be argued later on, the instability of this solution and its lack of unity are caused by the fact that the binocular rivalry interaction is weak (see below). Generally rivalry interactions are not weak. The monocular rivalry term changes the brightness - and darkness - probabilities by stimulus-onset, -offset and -movements. Once established the image is maintained by means of fluctuations, like blinks, changes in pupil size and eye movements which cause a strong rivalry at the contours. Brightness (darkness) signals fall upon regions where the antagonistic probability with respect to the monocular interaction (darkness or brightness) is greater than the non-antagonistic one. The effects of the rivalry are spread over the percept by the co-operative interaction. The range of this spread increases with the strength and duration of the input and the value of the antagonistic probability (see Sections 2.1.4.1 and 2.3.3). In the case of identical stimulation of both eyes the input from one eye falls greatly within the region where

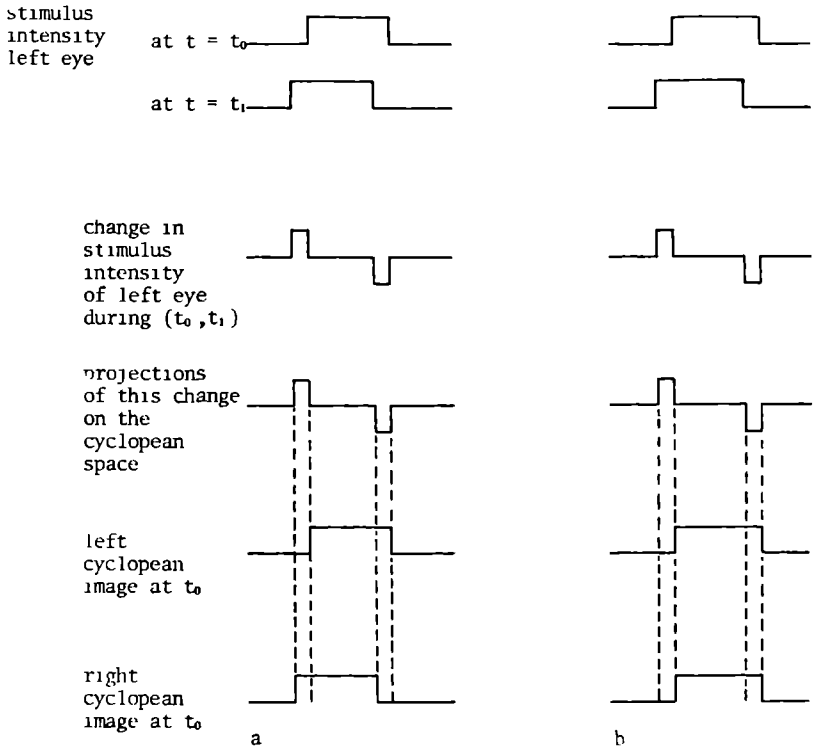


Figure 5.2.1. Input projection on the cyclopean image due to eye-movements during identical stimulation:
a. fusion is regained; b. fusion is lost.

the antagonistic probability with respect to the binocular interaction (same brightness type) is the greater one in the other eye (see Figure 5.2.1). If fusion is lost the input falls (for a greater part) outside this region. The latter causes a weak binocular interaction so that its influence can be neglected. There are two reasons for it. Firstly, there is hardly any spreading by the co-operative interaction, and secondly it is easily suppressed by the much stronger interaction in the

fused situation. In other words, the departure from the stationary solution - by fluctuations - is caused by a weak interaction, while the correction towards it, is caused by a strong interaction. In conclusion, the normal stimulus fluctuations disturb scarcely the stationary solution in the case of identical stimulation.

In the case of complementary stimuli these interaction strengths are reversed. Therefore (5.2.4) is not a stable solution. But the two solutions, in which the cyclopean image coincides with one of the two retinal images, are stable. It can be made clear by combining the probability densities in another way (see (5.2.5)). Let

$$\begin{bmatrix} RS(d,r,t) \\ RD(d,r,t) \\ LS(d,r,t) \\ LD(d,r,t) \end{bmatrix} = \begin{bmatrix} P(br,d,r,t) + P(dr,d,r,t) \\ P(br,d,r,t) - P(dr,d,r,t) \\ P(bl,d,r,t) + P(dl,d,r,t) \\ P(bl,d,r,t) - P(dl,d,r,t) \end{bmatrix} \qquad (5.2.5)$$

then (5.2.6) can be derived from (5.2.5) and the systems equation (2.3.25). Let

$$\begin{aligned} rf(r+d,t) &= \int dd' f(d+d')\, S_R(r-d',t) \\ \overline{rf}(r+d,t) &= \int dd' f(d+d')\, \{S_R(r-d',t)\} \\ rg(r-d,t) &= \int dd' g(d-d')\, S_R(r-d',t) \\ \overline{rg}(r-d,t) &= \int dd' g(d-d')\, \{S_R(r-d',t)\} \end{aligned} \qquad (5.2.7)$$

Analogously lf, $\overline{lf}$, lg and $\overline{lg}$ can be defined from $S_L(r-d',t)$. Apart from r, d and t equation (5.2.6) can be rewritten as in (5.2.8).

$$\Box \begin{bmatrix} RS \\ RD \\ LS \\ LD \end{bmatrix} = \int dd \left\{ \begin{bmatrix} \overline{rf}LS+rfLD \\ \overline{rf}LD+rfLS \\ \overline{lf}RS+lfRD \\ \overline{lf}RD+lfRS \end{bmatrix} - \begin{bmatrix} \overline{lf}RS+lfRD \\ \overline{lf}RD+lfRS \\ \overline{rf}LS+rfLD \\ \overline{rf}LD+rfLS \end{bmatrix} - 2 \begin{bmatrix} 0 \\ \overline{rg}RD-rgRS \\ 0 \\ \overline{lg}LD-lgLS \end{bmatrix} \right\} \qquad (5.2.8)$$

where $\Box$ represents the time derivation operation and the co-operative interaction. The (rivalry) terms at the right hand side equal zero, or

$$\frac{\partial}{\partial t}\int dd \begin{bmatrix} hS(d,r,t) \\ RD(d,r,t) \\ LS(d,r,t) \\ LD(d,r,t) \end{bmatrix} = \int dd \int dy \; \{ h(r-y,y) \begin{bmatrix} RS(d,y,t) \\ RD(d,y,t) \\ LS(d,y,t) \\ LD(d,y,t) \end{bmatrix} - H(y-r,r) \begin{bmatrix} RS(d,r,t) \\ RD(d,r,t) \\ LS(d,r,t) \\ LD(d,r,t) \end{bmatrix} \}$$

(5.2.6)

$$+ \int dd \int dd' \; f(d+d') \{ \begin{bmatrix} |S_R(r-d,t)|LS(-d',r,t) + S_R(r-d,t)LD(-d',r,t) \\ |S_R(r-d,t)|LD(-d',r,t) + S_R(r-d,t)LS(-d',r,t) \\ |S_L(r-d,t)|RS(\ d',r,t) + S_L(r-d,t)RD(\ d',r,t) \\ |S_L(r-d,t)|RD(\ d',r,t) + S_L(r-d,t)RS(\ d',r,t) \end{bmatrix}$$

$$- \begin{bmatrix} |S_L(r-d,t)|RS(\ d',r,t) + S_L(r-d,t)RD(\ d',r,t) \\ |S_L(r-d,t)|RD(\ d',r,t) + S_L(r-d,t)RS(\ d',r,t) \\ |S_R(r-d,t)|LS(-d',r,t) + S_R(r-d,t)LD(-d',r,t) \\ |S_R(r-d,t)|LD(-d',r,t) + S_R(r-d,t)LS(-d',r,t) \end{bmatrix} \}$$

$$-2\int dd \int dd' \; g(d-d') \{ \begin{bmatrix} 0 \\ |S_R(r-d,t)|RD(\ d',r,t) - S_R(r-d,t)RS(\ d',r,t) \\ 0 \\ |S_L(r-d,t)|LD(-d',r,t) - S_L(r-d,t)LS(-d',r,t) \end{bmatrix} \}$$

better, their strength is very small in comparison with the strength of the co-operative interaction term if

$$|rf| \equiv \overline{\overline{rf}}, \; |rg| \equiv \overline{\overline{rg}}, \; |lf| \equiv \overline{\overline{lf}}, \; |lg| \equiv \overline{\overline{lg}} \qquad (5.2.9a)$$

and

$$\frac{LD}{LS} \equiv \frac{-rf}{\overline{\overline{rf}}}, \; \frac{RD}{RS} \equiv \frac{-lf}{\overline{\overline{lf}}}, \; \frac{rg}{\overline{\overline{rg}}} \equiv \frac{-lg}{\overline{\overline{lg}}}. \qquad (5.2.9b)$$

The first four equations (5.2.9a) hold except in the direct neighbourhood of contrast. There the shape of the functions f and g causes the difference. However, the width of these functions is small ($\leq$ 6 min of arc; see Section 3.1.1) with respect to the sizes of the stimulus fields

(~1 deg of arc). The next two equations (5.2.9b) indicate that for each eye a part of the field is in a pure darkness or brightness state. Thus if brightness-left is not zero, darkness-left equals zero and vice versa. The same holds for the right eye. The last equation (5.2.9b) is the most interesting one because it expresses the requirement that the half-images have to be complementary (see (3.2.6)). It means that, if the stimuli are not of low intensity, the rivalrous interactions can be weak only if the stimuli are complementary.

The temporal behaviour of the system due to fluctuations can be clarified with the help of Figure 5.2.2. Only the interaction in the neighbourhood of contrast has to be studied, since (5.2.9a) holds everywhere except in such a region. Let

$$\begin{aligned} I &= \overline{\overline{rf}}LS + rfLD \\ II &= \overline{\overline{rf}}LD + rfLS \\ III &= \overline{\overline{lf}}RS + lfRD \\ IV &= \overline{\overline{lf}}RD + lfRS \\ V &= \overline{\overline{rg}}RD - rgRS \\ VI &= \overline{\overline{lg}}LD - lgLS \end{aligned} \qquad (5.2.10)$$

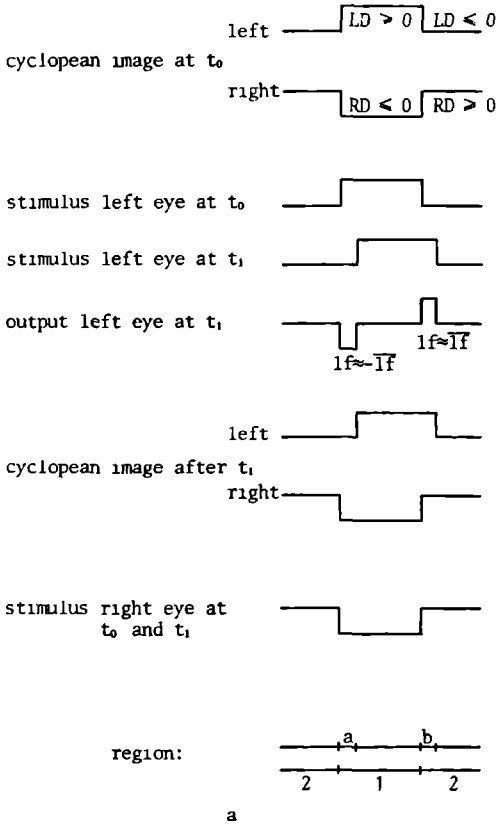
Because $|rf| \leq \overline{\overline{rf}}$, $|lf| \leq \overline{\overline{lf}}$, $|LD| \leq LS$ and $|RD| \leq RS$ it holds $I \geq 0$ and $III \geq 0$ for all r, d and t. In Figure 5.2.2b, which describes a regaining of fusion, it holds in region

$$\begin{aligned} &a: \quad IV \equiv -III \equiv \overline{\overline{lf}}(RD-RS) \\ &b: \quad IV \equiv III \equiv \overline{\overline{lf}}(RD+RS) \end{aligned} \qquad (5.2.11)$$

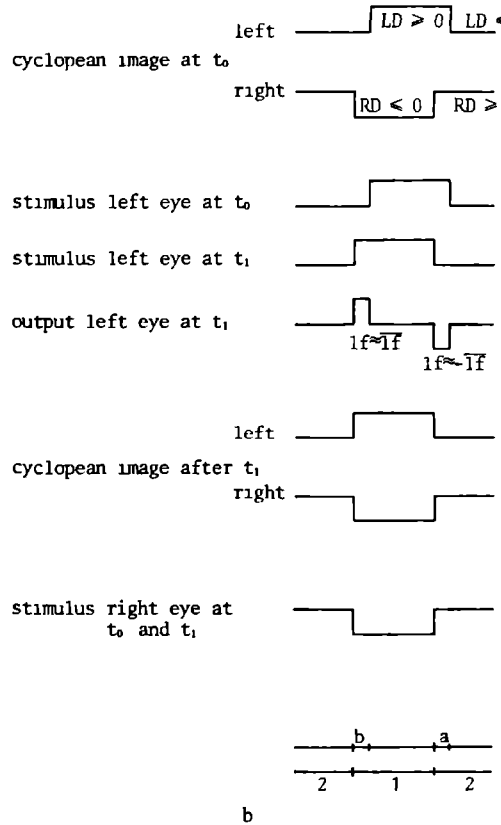
From equation (5.2.8) it follows that in both cases RS decreases and LS

See opposite page:

Figure 5.2.2. The interaction on the cyclopean image due to eye-movement during rivalry: a. fusion is lost; b. fusion is regained.


left
LD ≥ 0
LD ≤ 0
cyclopean image at t_0
right
RD ≤ 0
RD ≥ 0
stimulus left eye at t_0
stimulus left eye at t_1
output left eye at t_1
$1f \approx \overline{1f}$
$1f \approx -\overline{1f}$
left
cyclopean image after t_1
right
stimulus right eye at t_0 and t_1
region:
a
b
2
1
2

a


left
LD ≥ 0
LD ≤ 0
cyclopean image at t_0
right
RD ≤ 0
RD ≥ 0
stimulus left eye at t_0
stimulus left eye at t_1
output left eye at t_1
$1f \approx \overline{1f}$
$1f \approx -\overline{1f}$
left
cyclopean image after t_1
right
stimulus right eye at t_0 and t_1
b
a
2
1
2

b

increases, while RD increases in region a and decreases in region b. because RD is greater than, respectively smaller than zero in these regions, |RD| increases so that IV and III tend to zero. It means that a transition occurs as long as |RD| does not equal RS which is mostly the case. It can easily be seen from equation (5.2.12) which has been derived from equation (5.2.8) for region 1 where $RD \leq 0$ and $LD \geq 0$.

$$\square \begin{bmatrix} RS \\ RD \\ LS \\ LD \end{bmatrix} = \int dd \begin{bmatrix} \underline{rf}(LS-LD) - \underline{lf}(RS+RD) \\ -\underline{rf}(LS-LD) - \underline{lf}(RS+RD) - \underline{2rg}(RS+RD) \\ \underline{lf}(RS+RD) - \underline{rf}(LS-LD) \\ \underline{lf}(RS+RD) + \underline{rf}(LS-LD) + \underline{2lg}(LS-LD) \end{bmatrix} \qquad (5.2.12)$$

For region 2 left and right have to be reversed in equation (5.2.12) However, once a transition has started, RS decreases and RD increases so that the transition process stops itself. Because this transition process has a local character and stops itself, it causes an unstable perception along the complementary contrast but not over the whole field.

In Figure 5.2.2a, which describes a departure from fusion, (5.2.11) holds as in Figure 5.2.2b. From equation (5.2.8) it follows that in both cases RS decreases and LS increases, while RD increases in region a and decreases in region b. because RD is smaller than, or greater than zero respectively in these regions it tends to zero in both cases. It can easily be seen from equation (5.2.13) which has been derived from equation (5.2.8) for region a, where $LD \leq 0$ and $RD \leq 0$.

$$\square \begin{bmatrix} RS \\ RD \\ LS \\ LD \end{bmatrix} = \int dd \begin{bmatrix} \underline{rf}(LS-LD) - \underline{lf}(RS-RD) \\ -\underline{rf}(LS-LD) + \underline{lf}(RS-RD) - \underline{2rg}(RS+RD) \\ \underline{lf}(RS-RD) - \underline{rf}(LS-LD) \\ -\underline{lf}(RS-RD) + \underline{rf}(LS-LD) - \underline{2lg}(LS+LD) \end{bmatrix} \qquad (5.2.13)$$

For region b left and right have to be interchanged in the equation above. These fluctuations thus can change the dominance, in this case from the right to the left eye. However, they can have only a direct

influence along the complementary contrast.

The influences of this local transition on the whole field can be spread only by the co-operative interaction. However, it is doubtful if this stimulus situation as such is enough for a transition of the whole field. Surely it is state-dependent and the transition probability increases as the regions a and b and/or the input are larger. A transition can perhaps occur if fusion is lost by movement of the dominant eye after regaining fusion, or eye-movements during fusion, several times, since it will decrease the dominance of the dominant image a little each time.

Furthermore other fluctuations exist. A fluctuation that causes a transition over the whole field is a blink of one or both eyes. The process in the fused situations is the same as that in the regions a and b of Figure 5.2.2b, but now over the whole subfields 1 and 2 respectively. Thus a transition from one image to the other is slowed down by a decreasing of the source (see (5.2.12)): the difference between |RD| and RS or |LD| and LS. For every blink there is a transition from one image to the other, which goes from the stronger to the weaker one.

A blink can also occur during a non-fused situation. This is an interesting case. Then the process described above (see (5.2.12)) occurs over the part of the field in which the image remains complementary. Moreover, another process occurs in the part of the field where the images are not complementary. This process is analogous to that in the regions a and b of Figure 5.2.2a, but in this case it is not slowed down along the complementary contrast. If, for instance, the right eye dominates and if $RD \geq 0$ in the neighbourhood (see region 2) of such a non-complementary contrast (see region a), RD grows due to the interaction described above (5.2.12). However, because of the co-operative interaction this increase of RD is partly carried off to region a where it is transported to the other image. There is no other inhibition of this process. So this fluctuation can cause a total transition from the right to the left eye which always starts along the complementary contrast with a higher probability than in the case of non-fusion without blink. It is interesting to note that a statistical analysis by Levelt (1968)

of the rivalry dominance durations indicated towards a process in which it looks as if the transition is governed by blinks. On an average four blinks are necessary before a transition occurs.

In conclusion eye-movements cause a permanent unrest along the complementary contrast. Blinks and eye-movements in the fused situation cause a partial transition. Blinks and eye-movements in the nonfused situation cause a total transition, which always starts along the complementary contrast. Consequently a dominant image is reasonable stable, because the transition can occur only due to a special combination of fluctuations. But if a transition does occur, it is a complete one and due to the contrast.

5.3. The transition time

Naturally the fluctuations do not occur separately. Sometimes they act simultaneously and always in different order. They can weaken and strengthen each other. The speed of all these processes depends on the stimulus strength, due to contrast, of the nondominant eye because only the input from this eye is involved in the process as explained in the discussion above. The dynamic systems equation is a first-order equation which is linear in the input $S(r,t)$. Thus the speed of the process is roughly inversely proportional with $|S(r,t)|$, apart from the influences of the time characteristics of the fluctuations. It means that if a transition occurs from the right to the left image its duration is a decreasing function of $|S_L(r,t)|$. This duration is the dominance time of the right image. So it holds (see (5.1.2)) (h is a decreasing function)

$$\bar{t}_R = h'(|S_L(r,t)|)$$
$$\bar{t}_L = h'(|S_R(r,t)|) \qquad (5.3.1)$$

The quantities $|S_R(r,t)|$ and $|S_L(r,t)|$ fulfil the operational anchorages which Levelt attached to the stimulus strengths s_R and s_L

(see Section 5.1). Firstly $|S_R|$ and $|S_L|$ are decreasing functions of the angular distance from the contour in the test pattern (see Section 0.1.2). Secondly they are increasing functions of the physical contrast and account for non-linearity, contour sharpness and, if one so wishes, for retinal inhomogeneity. A test for this model can be done on the basis of Levelt's experiments on global rivalry. He measured durations of dominance as a function of physical contrast. He found that the dominance times $\bar{t}_R$ and $\bar{t}_L$ are proportional with $h(s_L)$ and $h(s_R)$ respectively (see (5.1.2)). Consequently the claim made by the model presented here is that s_L can be identified with $|S_L(r,t)|$ and s_R with $|S_R(r,t)|$. Equally the functions h and h' can be identified.

In Table 5.3.1 the mean image dominance and the alternation frequency per minute for three different stimuli have been represented. Because the function h is unknown the theoretical values for $1/|S_L|$ and $1/|S_R|$ are compared with Levelt's experimental findings. In the first and the third row these calculated values are of the order of the experimental values. In the second row this is not the case. Clearly, h is not the inverse function.

5.4. Other global rivalry phenomena

Most stimuli on global rivalry do not consist of half-images with complementary stimuli (see for instance Figure 1.1.1). However, the discusson of Section 5.2 holds generally for these patterns too. The only difference is that almost every fluctuation creates a situation as in the case of eye-movements or blinks during the absence of fusion. Consequently there is always a direct transition of one image to the other. These transitions occur locally independently if the stimuli are contrast-rich, as for instance in the case of Figure 1.1.1, because the influence of the co-operative interaction is broken down by the presence of contrast.

Three stimulus conditions in cd/m^2 (extra contrast in right stimulus).

left	710 and 85	100 and 85	100 and 12
right	110 and 5.75	110 and 5.75	110 and 5.75

Theoretical contrast is, following Section 0.1, proportional with $|S|$.

left	.379	.031	.473
right	.613	.613	.613

Mean dominance.

left	$1/\|S_R\|$	1.63	1.63	1.63
	exp. time $\bar{t}_L$ (sec)	2.62	2.59	2.56
right	$1/\|S_L\|$	2.64	32.00	2.11
	exp. time $\bar{t}_R$ (sec)	3.63	6.79	2.94

Alternation frequency.

$1/(1/\|S_R\| + 1/\|S_L\|)$	.231	.030	.267
exp. $1/(\bar{t}_L + \bar{t}_R)$	.160	.107	.182

Mean dominance left eye.

$(1/\|S_R\|)/(1/\|S_R\| + 1/\|S_L\|)$	.382	.048	.436
exp. $(\bar{t}_L)/(\bar{t}_L + \bar{t}_R)$	.419	.276	.466

See opposite page:

Table 5.3.1. Experimental results of Levelt (1968) on alternation frequency in binocular rivalry compared with theoretical predictions. $\frac{1}{|S|}$ is used instead of $h(|S|)$, since h is unkown.

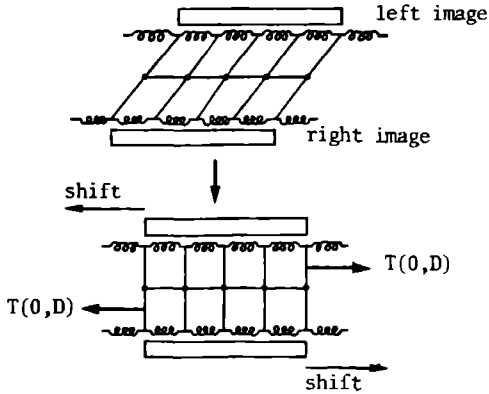


Figure 6.1.1. Transformation (6.1.3) is analogous with a translation of the input from both eyes.

6.1. One disparity difference

If both images are identical and have a disparity 2D, then apart from fluctuations the relation between the half-images L_R and L_L can be described by (6.1.1). It follows from Section 0.1 that the input of the cyclopean system can be described by (6.1.2). Analogously to (3.2.4) functions B and D can be defined (see (6.1.3)).

$$\begin{aligned} L_R(r-D,t) &= L(r,t) \\ L_L(r+D,t) &= L(r,t) \end{aligned} \qquad (6.1.1)$$

$$\begin{aligned} S_R(r,t) &= S(r+D,t) \\ S_L(r,t) &= S(r-D,t) \end{aligned} \qquad (6.1.2)$$

$$\begin{aligned} b(d,r,t) &= P(br,D+d,r+d,t) + P(bl,D-d,r+d,t) \\ D(d,r,t) &= P(dr,D+d,r+d,t) + P(dl,D-d,r+d,t) \end{aligned} \qquad (6.1.3)$$

- B and D are governed by an equation which is analogous to the equation in the monocular case (see (3.2.3)), to which another monocular term (see (3.2.5)) has been added. However, in this case the functions T(d,d') have been replaced by T(d,d'+D) (see Figures 6.1.1 and 1.2.12).

As in Section 3.2.3 it can be argued that the co-operative interaction and the monocular rivalry are independent of d, if b and D are zero almost everywhere and d lies outside the range of 1. Thus for the P(j,d,r,t) it holds that d equals D in the average. So the disparity is perceived and fusion occurs.

However, there is one complication. The term governed by T (see Figure 6.1.1) drives the d values of B and D towards -D. The success of the

driving is dependent on its strength. The strength increases with |D| (see Section 3.1.2). Thus there may exist some d_0 for which it holds that T drives d towards -D if $|D| > d_0$ (see also Sections 3.1.1, 3.1.2 and 1.2.6.4). d_0 depends on the type of stimulus. A stationary solution does not then exist because the binocular interaction cannot come to a solution. Clearly one has to do with a transient subsystem.

On the other hand, if the co-operative interaction and the rivalry terms are stronger than the influence of T (clearly if $|D| < d_0$) it follows that an absorbing subsystem exists for which the d values of b and b lie in the range of T. This range does not exceed stereo-acuity (see Section 3.1.2). However, it depends on the "initial state" whether a subsystem is an absorbing system or not (see Section 3.2). If the disparity difference between the initial state and the absorbing system (stationary solution) increases, the force of the binocular interaction term decreases, because the weighting function f decreases (see Section 3.1.1). Thus due to the other terms in the equation, solutions with disparity around -D can comprise another absorbing subsystem, so that the stationary solution, or rather the absorbing subsystem with disparities around 0, cannot be reached. However, if $|D| > d_0$, the influence of T can be stronger than the rivalry term. Then disparity is not observed and pure diplopia occurs. The features described here are not very interesting because the theory is based upon them. The characteristics of f, g and T can be chosen such that they will be realized.

6.2. Two disparity differences

Let a new pair of half-images be composed from a given pair of half-images, which give rise to the perception of disparity. The procedure is sketched in Figure 6.2.1 (see also Section 1.2.6.4). If I_R and I_L represent the original stimulus, the new stimulus is defined by

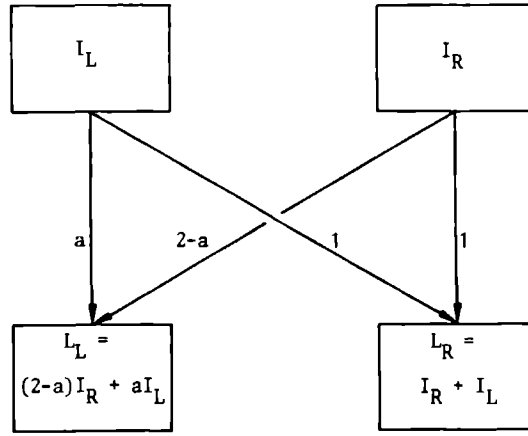


Figure 6.2.1. A stereogram constructed from another one.

$$\begin{aligned} L_R &= I_R + I_L \\ L_L &= (2-c)\, I_R + cI_L \end{aligned} \tag{6.2.1}$$

(6.2.2) can be deduced from (6.1.1) and (6.2.1).

$$\begin{aligned} L_R(r,t) &= L(r+D,t) + L(r-D,t) \\ L_L(r,t) &= (2-c)L(r+D,t) + cL(r-D,t) \end{aligned} \tag{6.2.2}$$

$$\begin{aligned} S_R(r,t) &= h(1)[S(r+D,t) + S(r-D,t)] \\ S_L(r,t) &= h(2-c)S(r+D,t) + h(c)S(r-D,t) \end{aligned} \tag{6.2.3}$$

In the case of sharp physical contrast it follows from Section 0.1 that S_R and S_L can be approximated as in (6.2.3), where h is some monotonously increasing function with h(0)=0. These S_R and S_L can be substituted in the systems equation. In order to get some insight in the induced process, one has to look at the following functions of d

$$\int dd'f(d-d')S_R(r-d')$$
$$\int dd'f(d-d')S_L(r+d') \qquad (6.2.4)$$

In Figure 6.2.2 several examples are given for the case that

$S_R^+(r+D)$ and thus, following (6.2.3), $S_R^+(r-D)$ show a local maximum. For reasons of simplicity S_R^+ is described as a delta function. The spread functions are the functions f. The first shows a situation in which the distance of the peaks of the spread functions is smaller that their spread (or half-width). The sum of the two spread functions adds up to a single peak function. Consequently this peak shifts as the relative height of both peaks changes. For the cyclopean system the input, as far as the disparity is concerned, looks like two identical half-images with a disparity that equals the distance between the new peaks of the left and the right image. Thus disparity detection will increase if this distance increases. It means that depth perception will increase if the ratio of the intensity of the two composing factors in the left image (see (6.2.2)) increases from 1 to infinity. However, as far as brightness is concerned (see Figure 6.2.3) the corresponding points of both half-images do not coincide with identical image points. They coincide in case the perceived disparity equals 2|D| or zero. Now a detector in a state br can point, for instance, towards a dr signal. Thus there will be depth perception but the brightness perception may be bad.

In the third column of Figure 6.2.2 the two spread functions add up to a clear double peaked function. There will then be an interaction between the peaks that have corresponding places. This leads to a disparity-less solution. A second interaction occurs between the remaining force of the largest peak in the right eye and the remaining force

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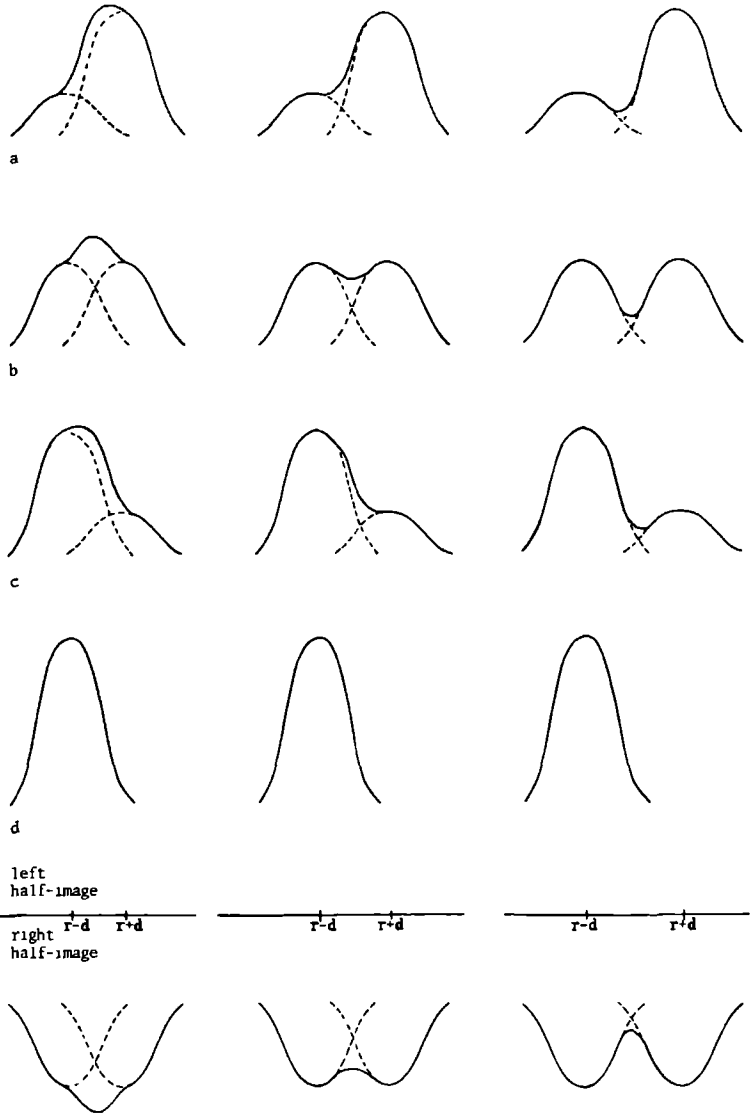
See opposite page:

Figure 6.2.2. Combined spread functions: a. c = .5; b. c = 1; c. c = 1.5; d. c = 2.


a
b
c
d
left
half-image
r-d
r+d
right
half-image
r-d
r+d
r-d
r+d

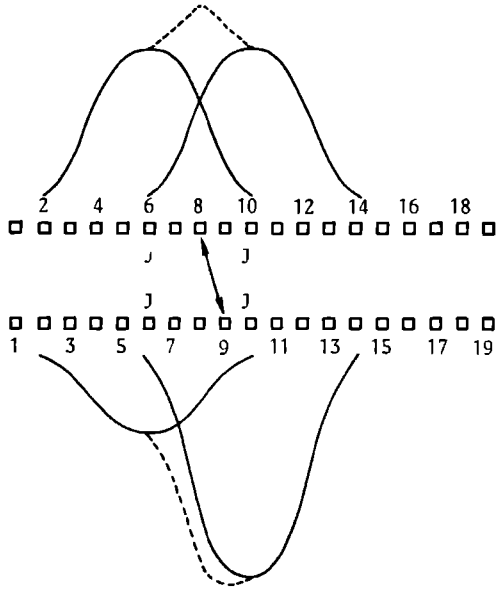


Figure 6.2.3. Net result of combined spread functions. The disparity connects points 8 and 9 instead of 6 and 6, 6 and 10, 10 and 6, or, 10 and 10. Thus the brightness input does not agree with the brightness state of the detector, if J_8 or J_9 does not equal J.

of the peak in the left eye which corresponds to the smallest one in the left eye. The latter interaction evokes a perception of depth. However, the peaks do not shift so the depth will be constant. Thus if the ratio of the intensities increases from 1 to infinity, from some value depth will be perceived. The depth is not affected by the value of this ratio. To a first order the total solution is a linear combination of this depth solution and the solution without depth. One will see a figure in depth eclipsed by the figure without depth. The second column of Figure 6.2.2 shows an intermediate case.

In all columns the inner tails of the combined spread functions have

a higher value than in the case of a single spread function. Consequently the width of a spread function in such a double combination is greater than normal. It means that depth can be perceived for larger 2|D| values than in the case of two single half-images. All characteristics, which can be deduced from the theory, have been observed by Kaufman et al. (1973). It has been argued in Section 1.2.6.4 that the revised formulation of the Julesz model could explain these observations too. In fact the revised model of Julesz coincides with the model presented here.

6.3. Some remarks

Figures 6.1.1, 6.2.2 and 6.2.3 show the property of the systems equation (2.3.25), that the location in the cyclopean field of (a part of) the percept can shift without a change in the location of (a part of) the stimulus. The percept lies somewhere between the two retinal locations. This displacement is called fusional displacement. Figures (6.2.2) and (6.2.3) show that the extent of the displacement depends on the retinal location and the relative strength of the retinal stimulation. In Section 1.2.2 it is argued, that one may not confuse fusional displacement, which is due to sensory fusion, and the displacement, which is caused by vergence or motor fusion.

In the discussion above the problem of the perception being attributable to a disparity difference has been avoided. In real stimuli the two half-images with some disparity have an identical surrounding without disparity (disparity zero) which evokes an impression with disparity zero. Both parts of the field will be solved as discussed above. The disparity difference between both parts determines the depth perception.

Naturally, two form-identical half-images with some disparity difference can also have a difference in intensity. Then the solution will be a solution with depth as above and brightness combination as discussed in Chapter 4. However, a discussion of such stimuli and solutions does

not add any new information to the discussion above.

For the sake of simplicity the disparity difference has been represented by a vector D throughout this Chapter. Depth can be perceived only along the horizontal axis. As it is argued in Section 3.1.1 the range of f is a circular disc. In order to solve this problem it is assumed (see Section 3.1.2) that the force due to T, which drives the disparity towards zero, along the y-coordinate is much stronger than this force along the x-coordinate. In the discussion above the disparity difference can therefore be read as a difference along the x-coordinate; thus D = (D,0).

A.1. The theory of Watson

Watson (1978) presented a formal theory of geometrical illusions and figural aftereffects. He conjectured that the visual space is a Riemann-space. If contrast is absent, the space is Euclidean, but if contrast is present, the geometry is non-Euclidean and the metric equation becomes

$$ds^2 = (1+h_1)dx^2 + (1+h_2)dy^2 \qquad (A.1)$$

where h_1 and h_2 represent the "force field". They are zero after adaptation to a contrast-free stimulus. Here only contrast in the x-direction will be discussed, so that h_2 equals zero by definition. h_1 is defined by

$$h_1(x) = \frac{\partial^2}{\partial x^2}\sqrt{pa}\int_{-\infty}^{\infty} dx' L(x') \exp[-(x-x')^2/am^2] \qquad (A.2)$$

where a and m are constants, p represents the number pi and L(x) represents the retinal stimulus. For the moment the question about retinal filters, eye-movements and binocular vision will be left out of consideration. If L(x) is the sinusoidal stimulus

$$L(x) = L(1+r\cos(fx+h)) \qquad (A.3)$$

then $h_1(x)$ becomes

$$h_1(x) = -pamf^2\cos(fx+h)\exp[-f^2am^2/4]\,Lr \qquad (A.4)$$

Thus with (A.1) it follows

$$ds^2 = (1-pamf^2 r \cos(ix+h)\ L \exp[-f^2 am^2/4])\ dx^2 \qquad (A.5)$$

Distances in the visual space can be calculated from (A.1) after the h_1 has been calculated. In the case of stimulus (A.3) a distance can be calculated from (A.5). Watson conjectured that if a subject is adapted to some stimulus, the "force field" of that stimulus is present, so that the metric of the space after adaptation is non-Euclidean even in the absence of stimulation. The part of the fields h_i, which is due to adaptation, can decay and change in sign over time. The fact that the space is non-Euclidean by adaptation causes the figural after-effects. The fact that a stimulus itself makes the space non-Euclidean causes the geometrical illusions.

A.2. A model for narrow-band spatial frequency channels

Here it is conjectured that the presence of a "force field", after adaptation to a sinusoidal grating, can cause the apparent presence of narrow-band spatial frequency channels; therefore three additional assumptions are made. Firstly, the perceived contrast is the stimulus contrast plus a "force field contrast". Secondly, the "force field contrast", which is the contrast after the attenuation of the stimulus by the "force field", is determined by the difference between the integrated intensities of a dark and a bright part, divided by twice the mean luminance. Thirdly, the threshold value of this quantity determines the visibility of stimulus contrast. The latter is a kind of modulation depth (see Section 2.2.1.1) after the attenuation. These assumptions imply that the perceived contrast of stimulus (A.6) is given by (A.9c).

$$L(x) = L_2(1+r_2\cos(gx)) \qquad (A.6)$$

The bright part of the "force field contrast", b(g), of (A.6) equals

$$b(g) = \int_{-p/2g}^{p/2g} dx'(1+h_1(x'))L(x') \qquad (A.7a)$$

and the dark part, d(g), equals

$$d(g) = \int_{-p/2g}^{p/2g} dx'(1+h_1(x'))L(x'+p/g) \qquad (A.7b)$$

(A.8b) follows from substitution of (A.6) in (A.8a), which is the definition of the "force field contrast". e is some weighting-constant.

$$c(g) = e\,\frac{b(g)-d(g)}{2L_2} \qquad (A.8a)$$

$$c(g) = e \int_{-p/2g}^{p/2g} dx'(1+h_1(x'))\, r_2\cos(gx') \qquad (A.8b)$$

Let C(g) be the perceived contrast of stimulus (A.6), then

$$C(g) = r_2 + c(g) \qquad (A.9a)$$

If the subject is adapted to (A.3), where h can be changed during the adaptation, then (A.9c) follows from (A.4), (A.8) and (A.9a,b)

$$s(f) = -epamf^2 Lr\cos(h)\exp[-am^2f^2/4] \qquad (A.9b)$$

$$C(g) = r_2[1 - \cos(fp/2g)[\frac{gf}{g^2-f^2}]s(f)] \qquad (A.9c)$$

If there was no adaptation it would hold that $s(f) = 0$ or $C(g) = r_2$. If r_1 is the modulation depth of stimulus (A.6) before adaptation then (A.10a) holds in the case of threshold measurements.

$$r_1 = r_2[1 - \cos(fp/2g)[\frac{gf}{g^2-f^2}]s(f)] \qquad (A.10a)$$

$$\frac{r_2-r_1}{r_1} = \frac{s(f)[\frac{gf}{g^2-f^2}]\cos(fp/2g)}{1-s(f)[\frac{gf}{g^2-f^2}]\cos(fp/2g)} \qquad (A.10b)$$

The right-hand side of equation (A.10b), which follows from (A.10a) fulfils the adaptation characteristics (see for instance Blakemore and Campbell, 1969). If f = g it holds that

$$\frac{r_2-r_1}{r_1} = \frac{ps(f)}{4-ps(f)} \qquad (A.11)$$

from which s(f) can be determined. Two typical curves of (A.10b) are shown in Figure A.1. The curves do not have the special characteristics shown in the experiments of Blakemore and Campbell. Some of their data are also shown in Figure A.1. The theoretical curves show a phase-shift to lower frequency values and a slower zero approach for large frequency values. However it is not intended to present a complete theory of narrow-band spatial frequency channels here. The purpose of this Appendix is only to show that a theory of narrow-band spatial frequency channels can be based upon a theory of local interaction. In the presentation above the medium-band attenuation filter is not incorporated; phase-shift is not supposed; the field evoked by the stimulus itself is not incorporated; some simple mechanism about contrast detection is assumed; the type of field-function (A.2) is open for discussion, and so on. Nevertheless a narrow-band can be calculated from the model which fits the halfwidth value. So with more adjusted assumptions a better fit with the data can surely be reached.

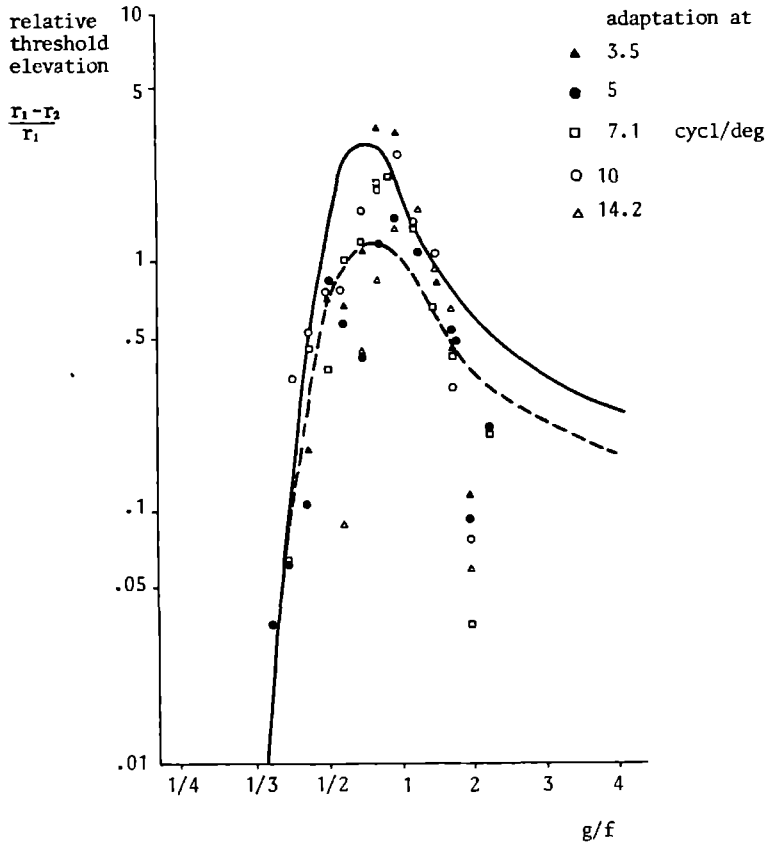


Figure A.1. Adaptation to spatial frequency. Data from Blakemore and Campbell (1969), subject FWC. The characteristics are normalized for spatial frequency. The data points have been shifted along the abscissa so that all adapting frequencies superimpose. For the theoretical curve ---- s(f)=2/p, for ____ s(f)=8/3p.

B.1. Clusters

Even if a percept is constant during some interval (t_1,t_2), noise causes fluctuations in the state of the system. Clearly the contributions of these states to the percept within this time-interval are perceptually equivalent. For this reason the class of states, which are perceptually equivalent, is introduced. It is called a cluster. It is the set of all states for which it holds that all percepts of two systems, of which the only difference is an interchange of the states x and y, are equal for every time t. It is said that the percept is invariant under the interchange of x and y. This invariance is denoted by $x \sim y$.

Such an equivalence class is not empty (see Section 2.3.1.1). The uncertainty of place implies that, for instance, an interchange of the states of two spatially neighbouring detectors which changes the cyclopean state from x into some y does not change the percept. Moreover saccadic movements and drift cause an uncertainty in the disparity of a binocular stimulus (Julesz, 1971, pp. 176-183). So, analogously to the uncertainty of place, it can be concluded that for an arbitrary detector there is, for every diparity d, an interval D(d) for which it holds that d and every d' in D(d) can be interchanged in a time-interval (t-dt,t) without changing the percept. So all states x' which differ from x by such a change in disparity belong to such a set. It is denoted by C(x).

The clusters are really equivalence classes, because $\sim$ is an equivalence relation. Firstly, (B.1) and (B.2) hold by definition.

$x \sim x$ (identity) (B.1)

x ~ y ---> y ~ x (reflexivity) (B.2)

Secondly, for every pair of states z and y in a cluster C(x) it holds that the percept is invariant under their interchange. This follows directly from the fact that the percept is invariant under the successive interchanges z and x, y and x, and, z and x. It is invariant under the net result, the interchange between z and y. Thus (B.3) holds. From (B.1), (B.2) and (B.3) it follows that ~ is an equivalence relation.

z ~ x x ~ y ---> z ~ y (transitivity) (B.3)

Two clusters, equivalence classes, are disjunct or equal. This can be shown as follows. Let the intersection of C(x) and C(y) be not empty, and, let z be an element of this intersection. Then it holds for every s in C(y), that the percept is invariant under the successive interchanges s and z, z and x, and, y and z. So it follows for every s in C(y) that s is in C(x). The reverse can be proved in the same way. Thus, if C(x) and C(y) are not disjunct, they are equal. On the basis of these clusters probability can be introduced in a way similar to methods in standard text books on statistical mechanics (see for instance Farquhar (1964)). A sketch will be given in the following sections.

B.2. The contribution of a cluster to a percept

Let $\underline{E}$ be a finite set of arbitrarily chosen cyclopean systems for which it holds that they all are governed by the same dynamic laws, receive the same input sequence and evoke the same reported percept at every time t. So the systems are identical in principle and their difference is only due to unnoticeable noise. Such a set is called an ensemble. It is supposed that the number of systems - $N(\underline{E})$ - in an ensemble is very high. Because of the uncertainty of place (and disparity) there will be a variability in the states of these systems at time t. However, instead of looking at the (set of) states one can look at

the (set of) clusters, which show much less variability.

The "presence" of a cluster during a period (t_0,t) is defined as the integrated time over all moments that the system is in a state belonging to this cluster. It is assumed that the presence of a cluster during a time interval $(t-dt,t)$ in which the percept at time t is established, determines its contribution to the percept. This assumption is based on the fact that if a percept is almost constant during the interval (t_1,t_2) with $dt/(t_2-t_1) \ll 1$, many different distributions of the individual states as well as of the clusters during several periods of length dt occur by noise. So only their presence can show constancy if dt is long enough with respect to the frequency of the noisy fluctuations and short enough with respect to the time difference between two moments of report about the percept.

Two states of the same cluster can be interchanged in an arbitrary period $(t-dt,t)$ without changing the percept at time t. So if a system is in $\underline{E}$, the system which is identical except for such an interchange can with equal chance be a member of the ensemble. Thus it follows that the number of cyclopean systems in $\underline{E}$ which are in a state belonging to the same cluster at a time t' in $(t-dt,t)$ will grow in the mean linearly with the growth of the presence of the cluster in $(t-dt,t)$, because $N(\underline{E})$ is very nigh. Or in other words, the ratio of the number of states of two clusters at an arbitrary time t' in $(t-dt,t)$ tends to the ratio of their presences in $(t-dt,t)$ if $N(\underline{E})$ tends to infinity. Thus a situation is arrived at which holds that the ratio of the contributions of two clusters to a percept at time t' can be estimated from the ratio of the number of systems, which are in a state belonging to a cluster, in an ensemble at time t' in $(t-dt,t)$. Because dt is much shorter than the difference between two moments of report about the percept, t' can be chosen arbitrarily. So t' will be chosen to be equal to t.

Furthermore the latter ratio has to do with the probability of finding a system in the ensemble in a state which belongs to the same cluster. Assumed that the dynamic laws of the system of a subject are time invariant and that it can be brought into a given "initial cluster" by, for instance, adaptation, then the system can be regarded during an experiment as a member of the ensemble $\underline{E}$. The other members of the ensemble are the same system for all possible repetitions of the experiment, and, - if the dynamic laws are subject-invariant - the systems involved in all possible repetitions with other subjects.

Let the subset $\mathbf{X}_t$ of $\mathbf{X}$ be the set of the states at time t of all systems in $\underline{E}$. Let the subset $\underline{E}_t(C)$ of $\underline{E}$ be the set of all systems in $\underline{E}$ for which the intersection of C and $\mathbf{X}_t$ is not empty, i.e. at time t some system is in a state belonging to C (see Figure B.1). Let $\underline{C}_t$ be the set of all clusters which have a state that belongs to $\mathbf{X}_t$. Then since a system is only in one state at a moment and clusters are disjunct or equal (see Section B.1), it holds for every two different clusters C' and C, both elements of $\underline{C}_t$ that

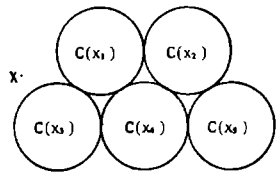
$$\underline{E}_t(C) \cap \underline{E}_t(C') = \emptyset \qquad (B.4)$$

$$\bigcup_{C \in \underline{C}_t} \underline{E}_t(C) = \underline{E} \qquad (B.5)$$

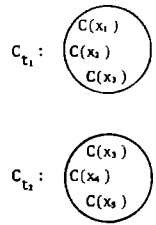
Let N(A) represent the number of elements of a finite set A. Then (B.6) holds for two finite sets A and B.

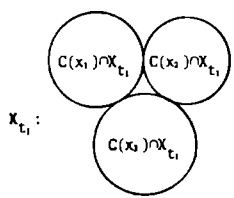
See opposite page:

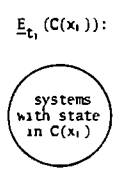
Figure B.1. Ven-diagrams of clusters and ensembles: a. set of states as a conjunction of five clusters: three present at time t_1 and three at t_2; b. the sets $\mathbf{X}_t$, $\underline{E}_t$ and $\underline{E}$, and their number of elements for $t=t_1$; c. idem for t_2.



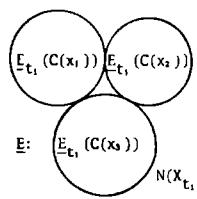
a







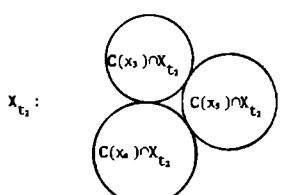
$\underline{E}_{t_1}(C(x_4)) = \emptyset$

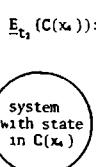


$N(X_{t_1}) = N(\underline{E}) = N$

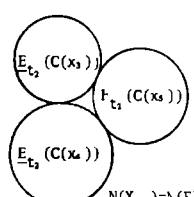
$N(\underline{E}_{t_1}(C(x_1))) = N(C(x_1) \cap X_{t_1})$

h





$\underline{E}_{t_2}(C(x_1)) = \emptyset$



$N(X_{t_2}) = N(\underline{E}) = N$

$N(\underline{E}_{t_2}((x_1))) = N(C(x_1) \cap X_{t_2}) = 0$

c

$$N(A \cup B) = N(A) + N(L) - N(A \cap L) \tag{B.6}$$

$$\sum_{C \in \underline{C}_t} N(\underline{E}_t(C)) = N(\underline{E}) \tag{B.7}$$

(B.7) follows from (B.4), (B.5) and (B.6) for every t. Let

$$r[C] = \frac{N(\underline{E}_t(C))}{N(\underline{E})} \tag{B.8}$$

Then (B.9) follows from (B.7) and (B.10) holds by definition.

$$\sum_{C \in \underline{C}_t} r[C] = 1 \tag{B.9}$$

$$N(\underline{E}_t(C)) = N(\emptyset) = 0 \quad \text{if } C \text{ is not an element of } \underline{C}_t \tag{B.10}$$

The ratio r[C(x)] can be interpreted as the probability of finding a system in a state of the cluster C(x) if the system is picked from the ensemble at a random time t. This is in fact what happens when an experimenter chooses a subject. The ratio of these probabilities tends to the ratio of the presences of the clusters if $N(\underline{E})$ tends to infinity. Together with the claims of Section B.2 it can be concluded that the contribution of a cluster to a percept can be calculated from the probability distribution as defined above.

B.4. Probability on X

In order to make calculations on X rather than on the set of clusters, a probability density function p(x) has to be defined which fulfils the requirement

$$\int_{y \in C(x)} p(y) = r[C(x)] \tag{B.11}$$

The notation

$\int_{y\in Y} g(y)$

with Y being a subset of X, stands for a combined summation and integration of the function g. The integration is carried out over the disparities d_i and the summation is carried out over the signal-types j_i for every detector at r_i (see (B.12)).

$$\int_{x\in Y} g(x) = \sum_{j_1}..\sum_{j_N} \int dd_1..\int dd_N\ g(d_1,j_1;..;d_N,j_N) \qquad (B.12)$$

Because the members of a cluster are psychophysically indistinguishable, it is also required that $p(y) = p(x)$ if $y \sim x$. The question whether a function p(x) exists and whether it is unique, depends on the clusters. Formally they have to be defined as subsets of X which fulfil special requirements. These have to fit in with the experimental observations about cluste s. It is assumed that the clusters, which will be defined below, fulfil the requirements.

The quantities d and j are in fact functions of r. The definition of a cluster is based on this property. A cluster, which is denoted by $C(f(r),d(r),j(r))$, is defined as the set of all states for which it holds that for every detector i the fraction of detectors j in (r_i, r_i+dr) for which the disparity d_j lies in the interval $(d(r_i), d(r_i)+dd(r_i))$ and the signal type is $j(r_i)$, equals $f(r_i)$. This definition satisfies the conditions due to the uncertainty of place and disparity (see Sections 2.2.2.1 and 2.3.1.2).

So the probability density on X can be defined as the function p of x in X for which holds

$$p(d(r_1),j(r_1);...;d(r_N),j(r_N))dd(r_1)...dd(r_N) = r[C(f(r),d(r),j(r))]$$

Thus a probability distribution has been defined on X and therefore x changes into a stochastic variable X.

C.1. Conservation of probability

A solution of the master-equation (see (2.3.8b) and (C.1)) is not a priori a probability density of X. In order to avoid confusion a general solution which is a continuous function of t, will be denoted by G(x,t). If a solution is supposed to be a probability density it is denoted by p(x,t).

$$\frac{\partial}{\partial t} p(x,t) = \int_{x' \in X} [w(x|x';t)p(x',t) - w(x'|x;t)p(x,t)] \qquad (C.1)$$

The requirements for G to be a probability density p of X are that $\int_{x \in X} p(x,t) = 1$ for all t and that $p(x,t) \geq 0$ for all x in X and all t. Below it will be shown that if G fulfils these requirements at some time t^* it fulfils them at all time $t \geq t^*$. Because the systems equation is a linear equation in t a general solution G has a free parameter for every x in X. It will be shown that these can be chosen such that G fulfils the requirements for a probability density at some time t^*. Thus by choosing appropriate initial conditions every general solution G can be converted into a probability density p. Let

$$A(x|x';t) = w(x|x';t) - \int_{x'' \in X} w(x''|x;t)\delta(x-x') \qquad (C.2)$$

by definition then the systems equation (C.1) can be rewritten as

$$\frac{\partial}{\partial t} p(x,t) = \int_{x' \in X} A(x|x';t)p(x',t). \qquad (C.3a)$$

and A has the property

$$\sum_{x\in X} A(x;x';t) = \sum_{x\in X} w(x;x';t) - \sum_{x''\in X} w(x'';x';t) = 0 \qquad \text{(C.3b)}$$

This property of A is called the conservation of probability because it follows that the total probability is conserved over time as will be shown below (see (C.7)).

Firstly, it follows for every solution G that g (see (C.4a)) is a constant, independent of t, since (C.4b) holds. Thus every solution G for which g does not equal zero can be normalized for all t (see (C.5)).

$$g = \sum_{x\in X} G(x,t) \qquad \text{(C.4a)}$$

$$\frac{d}{dt} \sum_{x\in X} G(x,t) = \sum_{x\in X} \sum_{x'\in X} A(x;x';t)G(x',t) = 0 \qquad \text{(C.4b)}$$

$$\sum_{x\in X} G^*(x,t) = \sum_{x\in X} \frac{G(x,t)}{G} = 1 \qquad \text{(C.5)}$$

Secondly, it follows for every solution G, that $G(x,t) \geq 0$ for all x in X and all $t \geq t^*$, if $G(x,t^*) \geq 0$ for all x in X. Because, let X_t^+, X_t^- and X_t^o be the sets of all x in X for which it holds respectively that $G(x,t) > 0$, $G(x,t) < 0$ or $G(x,t) = 0$. Let $\bar{X}_t^+$ be the conjunction of X_t^+ and X_t^o and let $\bar{X}_t^o$ be the set of all x in X_t^o with $\frac{\partial}{\partial t} G(x,t) \neq 0$. Then

$$\frac{d}{dt}\int_{x\in X_t^-} G(x,t) = \int_{x\in X_t^-} \frac{\partial G(x,t)}{\partial t} - \int_{x\in X_t^{-o}} G(x,t)\frac{\left[\frac{\partial}{\partial t}G(x,t)\right]}{\left|\frac{\partial}{\partial t}G(x,t)\right|} \tag{C.6}$$

$$= \int_{x\in X_t^-}\int_{x'\in X} A(x|x';t)G(x',t) + 0$$

$$= \int_{x\in X_t^-}\int_{x'\in X_t^{-+}} A(x|x';t)G(x',t) + \int_{x\in X_t^-}\int_{x'\in X_t^-} A(x|x';t)G(x',t)$$

$$= \int_{x\in X_t^-}\int_{x'\in X_t^{-+}} A(x|x';t)G(x',t) - \int_{x\in X_t^{-+}}\int_{x'\in X_t^-} A(x|x';t)G(x',t)$$

$$= \int_{x\in X_t^-}\int_{x'\in X_t^{-+}} w(x|x';t)G(x',t) - \int_{x\in X_t^{-+}}\int_{x'\in X_t^-} w(x|x';t)G(x',t)$$

$$\geq 0$$

for all t. (C.6) follows from the definitions and (C.3). It leads to the conclusion that

if $\int_{x\in X_{t^*}^-} G(x,t^*) = 0$, then $\int_{x\in X_t^-} G(x,t) = 0$ for all $t \geq t^*$,

or if $X_{t^*}^- = \emptyset$, then $X_t^- = \emptyset$ for all $t \geq t^*$.

From the two properties (C.5) and (C.6) it follows that for every solution $p(x,t)$ of the systems equation which is a probability density on X at t^*, it is a density at all $t \geq t^*$. This conservation of probability is expressed in formula (C.7).

$$\int_{x\in X} p(x,t) = \int_{x\in X} p(x,t^*) = 1 \tag{C.7a}$$

for all $t \geq t^*$ and

$$p(x,t) \geq 0 \tag{C.7b}$$

because $p(x,t^*) \geq 0$ for all x in X.

The system is a closed system. It means that there is no interaction with other systems apart from the system input which makes a change in A possible. However, if one looks at a subsystem, which can be defined by choosing a subset X_1 of X, then generally there will be an interaction between the subsystem defined by X_1 and another subsystem defined by another set X_2, subset of X. It means that there are an x in X_1, an x' in X_2 and a t in T such that $A(x'|x;t) \neq 0$ or $A(x|x';t) \neq 0$. Such a subsystem, defined by X_1, is called an open system. If $A(x'|x;t) = A(x|x';t) = 0$ for all x in X_1, all x' in X_2 and all t in T it is called a closed system. The conservation of probability holds for every closed system, because the proof, given above, is valid for it. Generally, every proof about the total system holds for a closed subsystem. Therefore every theorem for the total system holds mutatis mutandis for every closed subsystem. However, this is not true for open subsystems. So conservation of probability cannot be proved for them.

If X_1 defines an open system it is called a balanced, absorbing or transient system during a period (t^*,t') if for every solution p(x,t) the mean of

$$B_1(t) = \frac{d}{dt} \int_{x \in X_1} p(x,t) \qquad (C.8)$$

over this period equals, is greater or is smaller than zero, respectively. In the first case probability is conserved, in the second case it is absorbed from other subsystems until $B_1(t)$ reaches one. In the final case probability is lost to other subsystems until $B_1(t)$ reaches zero.

$G_1(x,t)$ is called an extended solution of the subsystem 1, if $G_1(x,t) = 0$ if x is not in X_1 and equals a solution $F_1(x,t)$ of the equation of the subsystem 1 if x in X_1. If G(x,t) is a solution of the systems equation and if $G(x,t) = 0$ for all x, not in X_1, it is an extended solution of the 1^{th} subsystems, because for all x in X_1 it holds

$$\frac{\partial}{\partial t} G(x,t) = \int_{x' \in X} A(x|x';t)G(x',t) = \int_{x' \in X_i} A(x|x';t)G(x',t) \qquad (C.9)$$

C.3. Stationary solutions

A stationary solution is a time-independent solution of the master equation (see (C.1)). For such a solution G(x) it holds that

$$\frac{\partial}{\partial t} G(x) = \int_{x' \in X} A(x|x';t)G(x') = 0 \qquad (C.10)$$

for all x in **X**. The conditions for the existence of a unique - apart from a constant - stationary solution can be found as follows. Suppose that at least two independent stationary solutions $G_1(x)$ and $G_2(x)$ exist. A third solution G(x) can be constructed as a linear combination of these two. Because G_1 and G_2 are independent there exist an x_0 in **X** and a y_0 in **X** such that (C.11) holds.

$$G_1(x_0)\, G_2(y_0) \neq G_1(y_0)\, G_2(x_0). \qquad (C.11)$$

$$G(x) = [G_2(x_0)+G_2(y_0)]G_1(x) - [G_1(x_0)+G_1(y_0)]G_2(x) \qquad (C.12a)$$

$$G(x_0) = -G(y_0) = G_2(y_0)G_1(x_0) - G_1(y_0)G_2(x_0) \neq 0 \qquad (C.12b)$$

For the solution G in (C.12a), (C.12b) holds. Thus if at least two independent solutions exist a third one, G, can be constructed such that its X_t^- and X_t^+ (see Section C.1) are non-empty sets. The index t in the notation of both sets will be dropped below because G(x) is a stationary solution.

$$\int_{x\in X^-}\int_{x'\in \bar{X}^+}[A(x|x';t)G(x') - A(x'|x;t)G(x)]$$

$$= \int_{x\in X^-}\int_{x'\in \bar{X}^+} A(x|x';t)G(x') - \int_{x'\in X^-}\int_{x\in \bar{X}^+} A(x|x';t)G(x') \qquad (C.13)$$

$$= \int_{x\in X^-}\int_{x'\in \bar{X}^+} A(x|x';t)G(x') + \int_{x\in \bar{X}^+}\int_{x'\in \bar{X}^+} A(x|x';t)G(x')$$

$$= \int_{x\in X^-}\int_{x'\in \bar{X}^+} A(x|x';t)G(x')$$

$$= 0$$

$A(x|x';t)G(x')$ and $-A(x'|x;t)G(x)$ are positive or zero for all pairs (x,x') in $X^- \times \bar{X}^+$ by definition. Formula (C.13), which follows from (C.10) and (C.3b), shows that both expressions are zero for all these pairs, since it implies that $A(x|x';t) = 0$ and $A(x'|x;t) = 0$ for all (x,x') in $X^- \times \bar{X}^+$, because $G(x') > 0$ and $G(x) < 0$. Furthermore, it follows from these results (see (C.14)) that $A(x'|x;t) = 0$ for all pairs (x,x') in $X^+ \times X^o$.

$$0 = \int_{x\in X} A(x'|x;t)G(x) \qquad (C.14)$$

$$= \int_{x\in X^+} A(x'|x;t)G(x) + \int_{x\in X^-} A(x'|x;t)G(x) + \int_{x\in X^o} A(x'|x;t)G(x)$$

$$= \int_{x\in X^+} A(x'|x;t)G(x)$$

The results about these zero transition probabilities, $A(x'|x;t) = 0$, are represented in Table C.1 for the case that X^o is not empty and for the case that it is empty. Thus transitions between states in X^- and X^+ do not occur. Because X^+ and X^- are not empty they define two mutually independent systems.

x' in	X^+	X^-	X^o
x in X^+		0	
X^-	0		
X^o	0	0	

a. $X^o \neq \emptyset$

x' in	X^+	X^-
x in X^+		0
X^-	0	

b. $X^o = \emptyset$

Table C.1. A priori zero transition probabilities $A(x|x';t)$.

Otherwise if A has for all t in (t^*,t') the properties which are represented in Table C.2a and C.2b where it is supposed that the X_i, subsets of X, are not empty, and X is, respectively, the conjunction of X_1 and X_2 or X is the conjunction of X_1, X_2 and X_3, at least two independent stationary solutions exist during this period. Because, let for i = 1,2 the function $F_1(x)$ with x in X_1 be a solution of (C.15) with x in X_1 and t in (t^*,t').

$$\int_{x' \in X_1} A(x|x';t)F_1(x') = 0 \qquad (C.15)$$

Then the functions $G_i(x)$ (i = 1,2) which are defined by $G_1(x) = F_1(x)$ if x in X_1 and $G_1(x) = 0$ otherwise are both solutions of equation (C.16) with x in X and t in (t^*,t'). The proof is trivial.

$$\int_{x' \in X} A(x|x';t)G(x') = 0 \qquad (C.16)$$

x' in	X_1	X_2
x in X_1		0
X_2	0	

a. reducible

x' in	X_1	X_2	X_3
x in X_1		0	
X_2	0		
X_3	0	0	

b. branching type

x' in X_1	X_2
x in X_1	
X_2	0

c. incompletely reducible

Table C.2. A priori zero probabilities A(x|x';t) with $t^* < t < t'$.

In conclusion, it is a necessary and sufficient condition for the existence of at least two independent stationary solutions that A has one of the two properties which are represented in Table C.2a and C.2b. Or in other words, a unique stationary solution - apart from other linearly dependent solutions - exists if and only if A is not one of these types. These proofs hold mutatis mutandis also for every subsystem which is defined by an arbitrary non-empty subset of X.

However in the discussion above the existence of a stationary solution is taken for granted. A proof for it can be given in the case that for every t the eigenvalue problem (C.17) can be solved in principle.

$$\int_{x' \in X} A(x|x';t)h(m_t,x') = m_t H(m_t,x) \qquad (C.17)$$

Such a proof is based on the conservation of probability (C.3b). This expression can be read as the proof for the existence of a left eigenfunction for eigenvalue zero from which the existence of a right eigenfunction for eigenvalue zero can be derived in a properly defined function-space. A sketch of the proof is as follows.

Let a function space over X with a norm be given. Let at every time t the eigenfunctions $E(n,m_t,x)$ - eigenvalue m_t - of A form a complete set. Then every function F in the space can be described as a linear combination of them. For every $E(n,m_t,x)$ with $m_t \neq 0$ (C.16) holds. It follows from (C.17) and (C.3b).

$$\int_{x \in X} E(n,m_t,x) = \frac{1}{m_t} \int_{x \in X} \int_{x' \in X} A(x|x';t)E(n,m_t,x') = 0 \qquad (C.18)$$

Thus if an eigenfunction for eigenvalue zero did not exist, or if for all eigenfunctions E(n,0,x) held

$$\int_{x \in X} E(n,0,x) = 0 \qquad (C.19)$$

then for every linear combination F - or in other words, for all functions F in the space - (C.20) would hold.

$$\int_{x \in X} F(x) = 0 \qquad (C.20)$$

For every F the function $|F|$, which is defined by $|F|(x) = |F(x)|$, belongs to the function-space because both have the same norm. Thus for every $|F|$ (C.20) would also hold, from which it follows that $|F(x)| = 0$ for almost every x in X. It means that all F in the space F(x) would equal zero for almost all x in X. This contradicts the assumption that the eigenvalue problem is solvable in principle. Thus at every time t

there exists an eigenfunction E(n,0,x) for which

$$\left| \sum_{x \in X} E(n,0,x) \right| \neq 0 \qquad (C.21)$$

Moreover it will be shown below that every system which has a stationary solution has at least one stationary solution G for which (C.22) holds and $G(x) \geq 0$ for all x in **X**. Such a solution can be normalized and interpreted as a probability density. So every (closed sub)system with solvable eigenvalue problem, (C.17), has at least one non-trivial stationary solution p(x); q.e.d.

$$\left| \sum_{x \in X} G(x) \right| \neq 0 \qquad (C.22)$$

C.4. Irreducibility and stationary solution

C.4.1. Definitions

If there exist an $\mathbf{X}_1$, an $\mathbf{X}_2$ and $\mathbf{X}_3$ such that A is one of the types described in Table C.2b and C.2c it is called respectively incompletely reducible or of a branching type during (t^*,t') except when a $\mathbf{X}_1$ and a $\mathbf{X}_2$ exist such that it is of the type in Table C.2a. Then it is called reducible during (t^*,t'). If A is neither reducible nor incompletely reducible during (t^*,t') it is called irreducible during (t^*,t').

If A is reducible during (t^*,t') every G(x,t) is a solution of the systems equation if and only if it is a linear combination

$$G(x,t) = m_1 G_1(x,t) + m_2 G_2(x,t) \qquad (C.23)$$

of extended solutions $G_j(x,t)$ of both closed subsystems which are defined by the $\mathbf{X}_j$ ($j = 1,2$). The proof is trivial. If A is incompletely reducible during (t^*,t') every G(x,t) is a solution of the systems

equation if and only if it is the sum $m(G_1(x,t)+G_2(x,t))$ of extended solutions $G_j(x,t)$ of both open subsystems which are defined by the X_j (see Table C.2c). The equations for both subsystems are the homogeneous equation (C.24a) for the x in X_2 and the non-homogeneous equation (C.24b) for the x in X_1.

$$\frac{\partial F_2(x,t)}{\partial t} = \int_{x' \in X_2} A(x|x';t)F_2(x',t) \qquad \text{(C.24a)}$$

$$\frac{\partial F_1(x,t)}{\partial t} = \int_{x' \in X_1} A(x|x';t)F_1(x',t) + \int_{x' \in X_2} A(x|x';t)F_2(x',t) \qquad \text{(C.24b)}$$

C.4.2. The system at infinity

The distinction between these different types of A is interesting for the behaviour of the solution G(x,t) of the systems equation if t tends to infinity. For the study of this behaviour a new system is introduced with the same set of states but with a probability transition function $\bar{A}(x|x';t)$ which equals $\lim[A(x|x';t')]$, if t' tends to infinity and if this limit exists, and equals A(x|x';t) otherwise. Thus $\bar{A}(x|x';t) = 0$ for all t if and only if $\lim[A(x|x';t)] = 0$, if t' tends to infinity; the type of A, if t tends to infinity, is fastened down in the type of $\bar{A}$. Then a solution G(x,t) of the systems equation tends to a solution $\bar{G}(x,t)$ of equation (C.25), if a t^* exists with a finite

$$M = \int_{x \in X} |G(x,t^*)|.$$

$$\frac{\partial \bar{G}(x,t)}{\partial t} = \int_{x' \in X} \bar{A}(x|x';t)\bar{G}(x',t) \qquad \text{(C.25)}$$

Proof: (C.27) follows from (C.6) and from the analogous equation (C.26).

$$\frac{\partial}{\partial t} \max_{x \in X_t^+} G(x,t) \leq 0 \qquad (C.26)$$

$$\max_{x' \in X} |G(x',t)| \leq \max_{x' \in X} |G(x',t^*)| = M \quad \text{for all } t \geq t^* \qquad (C.27)$$

Let $X^o(x)$ be the set of all x' in X for which $\lim[A(x|x';t')]$, if t' tends to infinity, exists. Let $X(x)$ be the set of x' in X with x' not in $X^o(x)$. Then (C.29) follows from (C.28) (see also (C.3a)).

$$\frac{\partial}{\partial t} G(x,t) = \sum_{x' \in X} A(x|x';t)G(x',t) \qquad (C.28)$$

$$= \sum_{x' \in X(x)} A(x|x';t)G(x',t) + \sum_{x' \in X^o(x)} A(x|x';t)G(x',t)$$

Since $A(x|x';t) - \bar{A}(x|x';t)$ in

$$\left|\frac{\partial}{\partial t} G(x,t) - \sum_{x' \in X} \bar{A}(x|x';t)G(x',t)\right| = \left|\sum_{x' \in X^o(x)} [A(x|x';t) - \bar{A}(x|x';t)]G(x',t)\right| \qquad (C.29)$$

approaches zero by definition if t tends to infinity, there exists for every number $e > 0$ a t_e so that for all $t > t_e$ (C.30) holds. It follows from (C.29) and (C.27).

$$\left|\frac{\partial}{\partial t} G(x,t) - \sum_{x' \in X} \bar{A}(x|x';t)G(x',t)\right| \leq \sum_{x' \in X^o(x)} |A(x|x';t) - \bar{A}(x|x';t)||G(x',t)| \qquad (C.30)$$

$$\leq e/M \sum_{x' \in X^o(x)} |G(x';t)| \leq e$$

Thus $\frac{\partial}{\partial t} G(x,t)$ approaches $\sum_{x' \in X} \bar{A}(x,x';t)G(x',t)$ if t tends to infinity, which means that G tends to a solution $\bar{G}$ (see (C.25)); q.e.d.

C.4.3. Reducibility and stationary solutions

If for a solution $G(x,t)$ X^-_t and X^+_t are both non-empty in case t tends to infinity, then $A(x|x';t)$ is reducible or of a branching type. The proof is as follows. From (C.6) and (C.26) it follows that X^-_t and X^+_t will become empty if t tends to infinity unless there is a t^* such that $X^-_{t^*}$ and $X^+_{t^*}$ are not empty and the derivatives with respect to t are zero for all $t \geq t^*$. Then (C.31) holds for all $t \geq t^*$ (see also (C.13)).

$$0 = \frac{\partial}{\partial t} \int_{x \in X^-_t} G(x,t) \qquad (C.31)$$

$$= \int_{x \in X^-_t} \int_{x' \in X^{-+}_t} A(x|x';t)G(x',t) - \int_{x \in X^{-+}_t} \int_{x' \in X^-_t} A(x|x';t)G(x',t)$$

Because $A(x|x';t) \geq 0$ for all (x,x') in $X \times X$ with $x \neq x'$ it follows that $A(x|x';t) = 0$ for all (x,x') in $X^-_t \times X^{-+}_t$ and all (x,x') in $X^{-+}_t \times X^-_t$. Similarly equation (C.31) holds if the + and - indices are interchanged. It implies that $A(x|x';t) = 0$ for all (x,x') in $X^+_t \times X^-_t$ and all (x,x') in $X^-_t \times X^+_t$. Thus if x in X^o_t then

$$\frac{\partial G(x,t)}{\partial t} = \int_{x' \in X^+_t} A(x|x';t)G(x',t) + \int_{x' \in X^-_t} A(x|x';t)G(x',t) = 0 \qquad (C.32)$$

This implies for all x in X that $G(x,t) = 0$ for all $t \geq t^*$ if $G(x,t^*) = 0$. Thus if $G(x,t')$ is not negative - $G(x,t') \geq 0$ - it cannot become negative - $G(x,t'') < 0$ - at a later time $t'' > t'$, because it is assumed that $G(x,t)$ is a continuous function of t (see Section C.1). Otherwise stated, $X^-_{t''}$ is a subset of $X^-_{t'}$ for all pairs (t'',t') with $t'' > t' \geq t^*$. There is at least one x in X which is an element of X^-_t for all $t \geq t^*$. Because if such an element did not exist, a pair (t'',t') with $t'' > t' \geq t^*$ would exist for which the intersection of $X^-_{t'}$ and $X^-_{t''}$ is empty. Then it would follow from X^-_t, subset of $X^-_{t''}$, subset of $X^-_{t'}$, if $t \geq t''$, that $X^-_t = \emptyset$ for all $t \geq t''$. It contradicts the assumption that, if t tends to infinity, $X^-_t \neq \emptyset$. If X^- is the set of all x in X with x in

$\bar{X}_t$ for all $t \geq 0$, then $\bar{X}_t$ tends to $\bar{X}$ which is not empty. Similarly it follows that such a non-empty $\bar{X}^+$ exists. From this it follows straightforwardly that $A(x|x';t)$ tends to a $\bar{A}(x|x';t)$ with $\bar{A}(x|x';t) = 0$ if (x,x') in $\bar{\bar{X}} \times \bar{X}^+$ or (x,x') in $\bar{X}^+ \times \bar{X}^-$. Thus $A(x|x';t)$ is reducible or of a branching type; q.e.d.

Every solution $G(x,t)$ tends to a solution $\bar{G}(x,t)$ *) of the subsystem which is generated by the conjunction of $\bar{X}^+$ and $\bar{X}^-$. This subsystem decomposes into two mutually independent subsystems which are generated by $\bar{X}^+$ and $\bar{X}^-$ if t tends to infinity. Consequently $G(x,t)$ tends to a sum of extended solutions $\bar{\bar{G}}^+(x,t)$ and $\bar{\bar{G}}^-(x,t)$ of both subsystems, where $\bar{\bar{G}}^+(x,t) \geq 0$ and $\bar{\bar{G}}^-(x,t) \leq 0$ for all x in X.

$$g^+ = \sum_{x \in X} \bar{\bar{G}}^+(x,t)$$
$$g^- = \sum_{x \in X} \bar{\bar{G}}^-(x,t) \qquad (C.33)$$

If g^+ and g^- (see (C.33)) are finite then $G(x,t)$ tends to a linear combination $g^+ p^+(x,t) + g^- p^-(x,t)$ of probability densities $p^+(x,t)$ and $p^-(x,t)$ (see (C.34)).

$$p^+(x,t) = \frac{\bar{\bar{G}}^+(x,t)}{g^+}$$
$$p^-(x,t) = \frac{\bar{\bar{G}}^-(x,t)}{g^-} \qquad (C.34)$$

If $G(x,t)$ is such that either $\bar{X}^+_t$ or $\bar{X}^-_t$ becomes empty with t tending to infinity, then $\bar{G}(x,t) \leq 0$ or $\bar{G}(x,t) \geq 0$, respectively, for all x in X, if t tends to infinity. Then, if g (see (C.4a)) exists, $G(x,t)$ tends to the probability distribution

*) extended if $\bar{X}^0 \neq \emptyset$

$$p(x,t) = \frac{\bar{G}(x,t)}{g} \qquad (C.35)$$

apart from a coefficient g. Furthermore if both X_t^+ and X_t^- become empty, G(x,t) tends to every probability distribution apart from a coefficient zero. In conclusion, every solution G(x,t) tends to a linear combination of probability densities $p_1(x,t)$ on X. Particularly this holds for every non-trivial stationary solution G(x). Then G can be normalized if either X^+ or X^- is empty; it follows from Section C.3 that if both X^+ and X^- are not empty also $\bar{G}^{-+}(x)$ and $\bar{G}^{--}(x)$ are stationary solutions, which both can be normalized. Consequently every system which has a stationary solution has at least one stationary solution which can be interpreted as a probability density.

C.4.4. Unique solutions for an irreducible system

If $\bar{A}(x|x';t)$ is not reducible or of a branching type it follows that either $G(x,t) \geq 0$ or $G(x,t) \leq 0$ for all x in X if t tends to infinity. If G(x,t) is not the trivial solution the normalized function G(x,t)/g can be regarded as a probability density. Thus apart from a factor g every solution G(x,t) is a probability density p(x,t). Particularly it holds, as follows from Section C.3, that a - non-trivial - unique normalized stationary solution $\bar{p}(x)$ exists.

Let X^o be the set of all x in X with $\bar{p}(x) = 0$. If X^o is not empty it generates a transient subsystem so that an arbitrary p(x,t) becomes zero for all x in X^o if t tends to infinity. It is shown as follows: Let X^* be the set of all x in X with x not in X^o then X^* is not empty and (C.36) follows using the conservation of probability (C.3b).

$$0 = \int_{x\in X^*} \int_{x'\in X} A(x|x';t)p(x') = \int_{x\in X^*} \int_{x'\in X^*} A(x|x';t)p(x') \qquad (C.36)$$

$$= -\int_{x\in X^o} \int_{x'\in X^*} A(x|x';t)p(x')$$

This can be true only if $A(x|x';t) = 0$ for all (x,x') in $X^o \times X^*$.

Therefore X^o generates a transient subsystem (see Table C.2c).

Proof: If x in X^o, (C.37), which follows from (C.36) and (C.5b), holds for an arbitrary p(x,t) - and also for an arbitrary G(x,t).

$$\frac{\partial}{\partial t}\int_{x\in X^o} p(x,t) = \int_{x\in X^o}\int_{x'\in X} A(x;x';t)p(x',t) = \int_{x\in X^o}\int_{x'\in X^o} A(x;x';t)p(x',t)$$

$$= -\int_{x\in X^*}\int_{x'\in X^o} A(x;x';t)p(x',t) \qquad (C.37)$$

because A is not reducible if t tends to infinity it follows that (C.37) is negative if t tends to infinity unless p(x,t) is zero for all x in X^o. If not, p(x,t) becomes zero for all x in X^o if t tends to infinity; q.e.d.

Moreover it follows for every non-trivial solution $G^*(x,t)$ of the subsystem generated by X^* that $G^*(x,t)/\bar{p}(x)$ becomes independent of x for all x in X^* if t tends to infinity. This means that every solution tends to the stationary solution apart from a factor. It is a very impotant theorem. Beceause it states that if a stationary solution of an irreducible system can be found, it is the only possible solution if t tends to infinity.

Proof: Let

$$g(x,t) = \frac{p^*(x,t)}{\bar{p}(x)} \qquad (C.38)$$

for all x in X^*. Then $g(x,t) \geq 0$ for all x in X^* and all t. The "entropy" S(t) of the subsystem, generated by X^*, is defined by

$$S(t) = -s\int_{x\in X^*} \bar{p}(x)g(x,t)\ln[g(x,t) + S_0] \qquad (C.39)$$

where s and S_0 are constants. It will be proven below that S(t) reaches a maximum, S_0, if t tends to infinity.

This proof that is as follows. (C.40) follows from (C.39) and (C.2).

$$\frac{dS}{dt} = -s \int_{x\in X^*} [1+\ln g(x,t)] \frac{\partial p^*(x,t)}{\partial t} \qquad (C.40)$$

$$= -s \int_{x\in X^*} \int_{x'\in X^*} [1+\ln g(x,t)][A(x|x';t)p^*(x',t)]$$

$$= -s \int_{x\in X^*} \int_{x'\in X^*} w(x|x';t)[[1+\ln g(x,t)]p^*(x',t)-[1+\ln g(x',t)]p^*(x',t)]$$

$$= -s \int_{x\in X^*} \int_{x'\in X^*} w(x|x';t)p(x')g(x',t)[\ln(\frac{g(x,t)}{g(x',t)})]$$

Since p(x) is a stationary solution, (C.41) follows from (C.10). Substitution of (C.41) in (C.40) delivers (C.42).

$$0 = \int_{x\in X^*} \int_{x'\in X^*} g(x,t)A(x|x';t)p(x') \qquad (C.41)$$

$$= \int_{x\in X^*} \int_{x'\in X^*} w(x|x';t)p(x')[g(x,t)-g(x',t)]$$

$$\frac{dS}{dt} = -s \int_{x\in X^*} \int_{x'\in X^*} w(x|x';t)p(x')g(x',t)[\ln(\frac{g(x,t)}{g(x',t)}) + 1 - \frac{g(x,t)}{g(x',t)}] \qquad (C.42)$$

The expression

$$g(x',t)[\ln(\frac{g(x,t)}{g(x',t)}) + 1 - \frac{g(x,t)}{g(x',t)}]$$

equals zero, if $g(x',t) = 0$ or if $g(x,t) = g(x',t)$ and is smaller than zero otherwise since $\ln(x) + 1 - x < 0$, if $1 > x \geq 0$. Because, if t tends to infinity, $g(x,t) \neq 0$ for some x in X^*, it follows that $\frac{dS}{dt} > 0$ unless $g(x,t) = 1$ for all x in X^*, which implies that S equals its maximum S_0. Thus S equals S_0 or approaches S_0 if t tends to infinity; q.e.d.

This maximum is reached if and only if $g(x,t)$ equals 1 for all x in X^*. This means that $G^*(x,t)$ equals or approaches $gp(x,t)$ for all x in X^* if t tends to infinity. Thus if $\bar{A}(x|x';t)$ is not reducible or of

a branching type every function $G(x,t)$ on $\mathbf{X}$ tends to $_{\mathrm{g}}\bar{p}(x)$.

C.4.5. Unique solutions at infinity

If $\bar{A}$ is reducible - as in Table C.2a - every solution $\bar{p}(x,t)$ equals a linear combination (see (C.43)) of extended solutions $\bar{p}_1(x,t)$ and $\bar{p}_2(x,t)$ of the two subsystems which are generated by $\mathbf{X}_1$ and $\mathbf{X}_2$, respectively. Condition (C.43b) expresses the conservation of probability (see (C.3b)).

$$\bar{p}(x,t) = \sum_{i=1}^{2} m_i \bar{p}_i(x,t) \qquad (C.43a)$$

$$\sum_{i=1}^{2} m_i = 1 \qquad (C.43b)$$

If a transient subsystem does not exist, the transition probability $\bar{A}$ of the subsystems which are generated by the $\mathbf{X}_i$ are reducible or irreducible. In the first case $\bar{p}_i(x,t)$ also equals a linear combination of extended solutions; in the second instance such a combination does not exist. Thus such a bifurcation procedure (C.43) can be continued until every such subsystem has an irreducible $\bar{A}$. It can be done only if the number of these subsystems is denumerable. Then $\bar{p}(x,t)$ equals a linear combination (see (C.43c))

$$\sum_{j=1}^{n} m_j \bar{p}_j(x,t) \qquad (C.43c)$$

of extended solutions $\bar{p}_j(x,t)$ of subsystems with an irreducible $\bar{A}$; n may tend to infinity. It holds for the extended stationary solutions $\bar{p}_j(x)$ of these subsystems that $\bar{p}_j(x) > 0$ for all x in $\mathbf{X}_j$ and equals zero otherwise. Every stationary solution $\bar{p}(x)$ of the system equals a linear combination as in (C.43d).

$$p(x) = \sum_{i=1}^{n} m_i \bar{p}_i(x) \qquad \text{(C.43d)}$$

If the subsystems which are generated by the X_j have irreducible $\bar{A}$ then (C.43c) approaches (C.43d) if t tends to infinity, since every $\bar{p}_j(x,t)$ approaches $\bar{p}_j(x)$.

Similar arguments hold for an arbitrary solution G(x,t). In Section (C.4.3) it is shown that G(x,t) tends to a linear combination - $g\bar{p}(x,t)$, or $g^+\bar{p}^+(x,t) + g^-\bar{p}^-(x,t)$ - of one or two subsystems which are generated by X^+ and/or X^-. It holds that $\bar{p}^+(x,t) > 0$ if x in X^+ and equals zero otherwise, $\bar{p}^-(x,t) > 0$ if x in X^- and equals zero otherwise and either $\bar{p}(x,t) > 0$ if x in X^+ and equals zero otherwise or $\bar{p}(x,t) > 0$ if x in X^- and equals zero otherwise. Since $\bar{p}^+(x,t)$, $\bar{p}^-(x,t)$ as well as $\bar{p}(x,t)$ can be written as a linear combination of extended solutions $\bar{p}_j(x,t)$ of subsystems with an irreducible $\bar{A}$, the arguments above holds for every solution G(x,t).

From (4.4.2) and (4.4.5) follows

$$s_h = 1+m+n$$
$$s_L = 1+m-n \qquad (D.1)$$

Below sum- and difference-functions of the probability distribution functions are introduced which cover the greater part of the stimulus-strength and make some perturbation-approach possible for small values of m or |n|. The systems equation is invariant under a transformation of n into -n and a simultaneous exchange of

$$\begin{bmatrix} P(br,d,r,t) \\ P(dr,d,r,t) \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} P(bl,-d,r,t) \\ P(dl,-d,r,t) \end{bmatrix}$$

Consequently their difference tends to $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$ if n equals zero and t increases to infinity. Since the solution for n = 0 has been known - identical stimulation of both eyes (see Section 3.2.3)- it can be assumed below that $n \neq 0$. It is argued in Appendix C that for the description of the behaviour of the system if t tends to infinity it suffices that one looks only for those solutions which have for all t the above mentioned properties. Therefore the following transformation (see (D.2)) is a general one with respect to the description of the stationary behaviour of the system.

$$\begin{aligned} P(br, d,r,t) &= (1/2)[(1+m)SB(d,r,t) + nVB(d,r,t)] \\ P(dr, d,r,t) &= (1/2)[(1+m)SD(d,r,t) + nVD(d,r,t)] \\ P(bl,-d,r,t) &= (1/2)[(1+m)SL(d,r,t) - nVB(d,r,t)] \\ P(dl,-d,r,t) &= (1/2)[(1+m)SD(d,r,t) - nVD(d,r,t)] \end{aligned} \qquad (D.2)$$

Substitution in the systems equation (2.3.25) delivers the equation

$$\frac{\partial}{\partial t}\begin{bmatrix} SB(d,r,t) \\ VB(d,r,t) \\ SD(d,r,t) \\ VD(d,r,t) \end{bmatrix} = \int dr' \{ R(r-r',r') \begin{bmatrix} SB(d,r',t) \\ VB(d,r',t) \\ SD(d,r',t) \\ VD(d,r',t) \end{bmatrix} - R(r'-r,r) \begin{bmatrix} SB(d,r,t) \\ VB(d,r,t) \\ SD(d,r,t) \\ VD(d,r,t) \end{bmatrix} \}$$

$$+ \int dd' \{ T(d-d',d') \begin{bmatrix} SB(d',r,t) \\ VB(d',r,t) \\ SD(d',r,t) \\ VD(d',r,t) \end{bmatrix} - T(d'-d,d) \begin{bmatrix} SB(d,r,t) \\ VB(d,r,t) \\ SD(d,r,t) \\ VD(d,r,t) \end{bmatrix} \} \qquad (D.3)$$

$$- \int dy \frac{f(d+y)}{2} \begin{bmatrix} S^+(r+y)[SB(d,r,t)-VB(d,r,t)] - S^+(r+d)[SB(y,r,t)-VB(y,r,t)] \\ S^+(r+y)[SB(d,r,t)-VB(d,r,t)] + S^+(r+d)[SB(y,r,t)-VB(y,r,t)] \\ S^-(r+y)[SD(d,r,t)-VD(d,r,t)] - S^-(r+d)[SD(y,r,t)-VD(y,r,t)] \\ S^-(r+y)[SD(d,r,t)-VD(d,r,t)] + S^-(r+d)[SD(y,r,t)-VD(y,r,t)] \end{bmatrix}$$

$$- \int dy \frac{g(d-y)}{2} \begin{bmatrix} S^-(r+y)[SB(d,r,t)+VB(d,r,t)] - S^+(r+d)[SD(y,r,t)+VD(y,r,t)] \\ S^-(r+y)[SB(d,r,t)+VB(d,r,t)] - S^+(r+d)[SD(y,r,t)+VD(y,r,t)] \\ S^+(r+y)[SD(d,r,t)+VD(d,r,t)] - S^-(r+d)[SB(y,r,t)+VB(y,r,t)] \\ S^+(r+y)[SD(d,r,t)+VD(d,r,t)] - S^-(r+d)[SB(y,r,t)+VB(y,r,t)] \end{bmatrix}$$

$$-m \int dy \frac{f(d+y)}{2} \begin{bmatrix} S^+(r+y)[SB(d,r,t)+VB(d,r,t)] - S^+(r+d)[SB(y,r,t)+VB(y,r,t)] \\ S^+(r+y)[SB(d,r,t)-VB(d,r,t)] + S^+(r+d)[SB(y,r,t)-VB(y,r,t)] \\ S^-(r+y)[SD(d,r,t)+VD(d,r,t)] - S^-(r+d)[SD(y,r,t)+VD(y,r,t)] \\ S^-(r+y)[SD(d,r,t)-VD(d,r,t)] + S^-(r+d)[SD(y,r,t)-VD(y,r,t)] \end{bmatrix}$$

$$-m \int dy \frac{g(d-y)}{2} \begin{bmatrix} S^-(r+y)[SB(d,r,t)-VB(d,r,t)] - S^+(r+d)[SD(y,r,t)-VD(y,r,t)] \\ S^-(r+y)[SB(d,r,t)+VB(d,r,t)] - S^+(r+d)[SD(y,r,t)+VD(y,r,t)] \\ S^+(r+y)[SD(d,r,t)-VD(d,r,t)] - S^-(r+d)[SB(y,r,t)-VB(y,r,t)] \\ S^+(r+y)[SD(d,r,t)+VD(d,r,t)] - S^-(r+d)[SB(y,r,t)+VB(y,r,t)] \end{bmatrix}$$

From (D.2) and (4.4.4) it follows that $m^2+n^2=1$.

If m is small, a perturbation approach can be used to solve the equation. Then SB, SD, VB and VD are developed as a series of powers of m.

These series are substituted in equation (D.3) and for every power of m the concerning equation can be solved. Such a series can be formulated as in (D.4), and similarly for SD, VB and VD. It follows from the substitution that in the case of a stationary solution (D.5) holds.

$$SB(d,r,t) = \sum_{p=0}^{\infty} m^p \, SB_p(d,r,t) \qquad (D.4)$$

$$\begin{aligned} SB_0(d,r,t) &= VB_0(d,r,t) \\ SD_0(d,r,t) &= VD_0(d,r,t) \end{aligned} \qquad (D.5)$$

Furthermore

$$\begin{bmatrix} SB_0(d,r,t) \\ SD_0(d,r,t) \end{bmatrix}$$

is a solution of the equation for monocular stimulation (see Section 3.2.2).

If |n| is small, a perturbation approach can be used, which develops to n. Then SB can be expressed as (D.6)

$$SB(d,r,t) = \sum_{p=0}^{\infty} n^p \, \overline{\overline{SB}}_p(d,r,t) \qquad (D.6)$$

Similar formulations can be given for SB, VB and VD. Substitution in equation (D.3) shows that

$$\begin{aligned} \overline{\overline{SB}}_{2p} &= \overline{\overline{SB}}_{2p+1}, \\ \overline{\overline{SD}}_{2p} &= \overline{\overline{SD}}_{2p+1}, \\ \overline{\overline{VB}}_{2p} &= \overline{\overline{VB}}_{2p+1}, \\ VD_{2p} &= VD_{2p+1} \end{aligned} \qquad (D.7)$$

for all p. Furthermore it shows that

$$\begin{bmatrix} \overline{\overline{SB}}_0(d,r,t) \\ SD_0(d,r,t) \end{bmatrix}$$

is a solution of the equation for identical stimulation (see Section 3.2.3). It is difficult to describe the pair of solutions

$$\begin{bmatrix} \overline{\overline{VB}}_0(d,r,t) \\ \overline{\overline{VD}}_0(d,r,t) \end{bmatrix}$$

However, as will be shown later on, it is not important.

For both perturbation approaches the requirement

$$\int dd \sum_j P(j,d,r,t) = 1 \qquad (D.8)$$

determines the normalization of SB, VB, SD and VD. It follows that

$$\int dd\, [SB(d,r,t) + SD(d,r,t)] = \frac{1}{1+m} \qquad (D.9)$$

Following Chapter 3 in the case of monocular stimulation as well as the case of identical binocular stimulation, continuity of the distribution functions $P(j,d,r,t)$ - and their linear combinations - in the R^2_d and the R^2_c spaces is supposed. Consequently the maximum input signals are independent of the disparity d; they will be denoted by $max(j,r,t)$ with $j = br$ etc, thus

$$max(j,r,t) = \max_{(r',t') \in V(br,d,r,t)} (S^+_R(r'+d,t')) \qquad (D.10)$$

As it has been argued in Chapter 3, the perceived disparity is 0 with some region of uncertainty, the distribution of which is described by the $P(j,d,r,t)$ as function of the disparity. The perceived total brightness $B(r,t)$ is the sum of all brightness in this region (see (4.2.2)). Thus

$$B(r,t) = \int dd\, B(d,r,t) \qquad (D.11)$$

Let $e_j = 1$ if $j = br$ or $j = bl$ and let $e_j = -1$ if $j = dr$ or $j = dl$. Then

it follows with (D.10) and (D.11) that

$$B(r,t) = \int dd\, B(d,r,t) = \int dd \sum_j e_j \max(j,r,t) P(j,d,r,t) \qquad (D.12)$$

This expression can be reformulated as a function of s_R and s_L by introducing the functions in (D.13). Then (D.12) transforms into (D.14).

$$\max(+,r,t) = \frac{2\max(br,r,t)}{s_R} \qquad (D.13)$$

$$\max(-,r,t) = \frac{2\max(dr,r,t)}{s_L}$$

$$B(r,t) = \frac{1}{2}\int dd\{ \max(+,r,t)[s_R P(br,d,r,t) + s_L P(bl,d,r,t)] \qquad (D.14)$$
$$-\max(-,r,t)[s_R P(dr,d,r,t) + s_L P(dl,d,r,t)] \}$$

Using (D.1) and (D.2), (D.14) can be expressed in terms of the functions SD, VB, SD and VD as follows

$$B(r,t) = \frac{1}{2}\int dd\{ \max(+,r,t)[(1+m)^2 SB(d,r,t) + n^2 VB(d,r,t)] \qquad (D.15)$$
$$-\max(-,r,t)[(1+m)^2 SD(d,r,t) + n^2 VD(d,r,t)]$$

Substitution of the solution for small m (D.5) and using a normalized expression (D.9) for the SB etc., delivers

$$B(r,t) = \frac{\int dd[\max(+,r,t) SB_0(d,r,t) - \max(-,r,t) SD_0(d,r,t)]}{2\int dd[SB_0(d,r,t) + SD_0(d,r,t)]} * \frac{(1+m)^2 + n^2}{1+m}$$

or in shortened form, where K(r,t) is defined implicitely,

$$B(r,t) = K(r,t)\frac{(1+m)^2 + n^2}{1+m} \qquad (D.16b)$$

Similarly an expression for small n can be obtained

$$B(r,t) = \frac{\int dd[\max(+,r,t)\bar{\bar{SB}}_0(d,r,t) - \max(-,r,t)\bar{\bar{SD}}_0(d,r,t)]}{2\int dd[\bar{\bar{SB}}_0(d,r,t) + \bar{\bar{SD}}_0(d,r,t)]} * \frac{(1+m)^2(1+n)}{(1+m)(1+n)}$$

$$+ \frac{\int dd[\max(+,r,t)\bar{\bar{VB}}_0(d,r,t) - \max(-,r,t)\bar{\bar{VD}}_0(d,r,t)]}{2\int dd[\bar{\bar{SB}}_0(d,r,t) + \bar{\bar{SD}}_0(d,r,t)]} * \frac{(1+n)\,n^2}{(1+m)(1+n)}$$

or in shortened form, where $\bar{K}(r,t)$ and $\bar{L}(r,t)$ are defined implicitely,

$$B(r,t) = \bar{K}(r,t)(1+m) + \frac{n^2}{1+m}\bar{L}(r,t) \qquad (D.17b)$$

From (4.4.4) and (D.1) it follows $m^2+n^2 = 1$, so that for small m

$$B(r,t) = 2K(r,t) \qquad (D.18a)$$

and for small n

$$B(r,t) = 2\bar{K}(r,t) + \frac{n^2}{1+m}[\bar{L}(r,t) - \bar{K}(r,t)] \qquad (D.18b)$$

The functions K, $\bar{K}$ and $\bar{L}$ are independent of the relative input strength s. The latter term of (D.18b) contains the unknown solution

$$\begin{bmatrix} \bar{\bar{VB}}_0(d,r,t) \\ \bar{\bar{VD}}_0(d,r,t) \end{bmatrix}$$

Knowledge of this solution is not important, since this term is of the order n^2. It can be neglected.

Physical quantities

R^2_L, R^2_R	retinal space
r	space co-ordinate
T	time space
t	time
D,d	stimulus disparity
L	Luminance

Perceptual quantities

S^+	retinal output, cyclopean input: stimulus onset
S^-	retinal output, cyclopean input: stimulus offset
r	place co-ordinate
d,D	perceptual disparity
J	set of signal types
j^c	signal type
B	brightness
R^2_c	cyclopean space
R^2_d	perceptual disparity space

Probability

$\underline{f}$	cyclopean field function
$\underline{\underline{f}}$	generalized cyclopean field function
$X_1 = R^2_d \times J_c$	state space detector i
X	state space cyclopean system
C[]	cluster, perceptual equivalence class
r[]	probability on cluster space
p	probability on state space cyclopean system
w	transition probability
r	transition probability due to co-operative interaction
s	transition probability due to other interactions

Systems equation

P	probability density function over cyclopean space
R	co-operative interaction
T	autonomous disparity interaction
f	range of binocular rivalry
g	range of monocular rivalry

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Summary

In this thesis a theory is developed which unifies models on visual phenomena, known as binocular rivalry or suppression, sensory fusion and disparity-depth perception. As appears from the literature several authors have conjectured that these phenomena originate from one system which is called here the cyclopean system.

The theory comprises qualitative as well as quantitative aspects. The latter are not only determined by the cyclopean system but also by stimulus transformations in the retinae. In Chapter 0 it is supposed that the retinal system is an algebraic non-linear transformation of the stimulus-intensity L followed by a spatio-temporal linear transformation. The former one is important with respect to the quantitative predictions of the theory on the cyclopean system. It is argued that this transformation is a quasi-logarithmic one - $F_2(L)$. because it can account for Fechner's law as well as Stevens' law.

Moreover an analysis of the spatial part of the linear transformation deliver also arguments in favour of $F_2(L)$. Elsewhere (Buffart, 1978) the spatial part was analysed as a contrast-blurring and a contrast-sharpening mechanism. The latter can be seen as a correction on the former. Their Fourier-transforms are the first two members of the set of orthogonal Laguerre-functions. It appears that the combination of $F_2(L)$ and this spatial mechanism leads to an explanation of some non-linear phenomena of the perception of sinusoidal gratings. Similarly, a non-linear effect in temporal attenuation-curves of the visual (retinal) system can be understood from $F_2(L)$.

The output of the retinal system depends on temporal changes of the input such that in the case of an abrupt spatial change of stimulus-

intensity, from L' into L, the output is proportional with $|F_2(L') - F_2(L)|$. Eye-movements cause a similar output along sharp physical contrast in the stimulus.

Some models on binocular suppression, sensory fusion and disparity-depth perception are discussed in Chapter 1. Sperling (1970) formulated a qualitative theory for the cyclopean system which comprises a fusion model and a suppression model. It is shown that this theory is falsified by experimental results of Levelt (1968). General objections against fusion theories and suppression theories are discussed. It is shown that phenomenologically observed suppression can be explained by a mechanism in which at micro-level a signal from one eye locally supports the influence of this eye instead of a mechanism in which it depresses the influence of the other eye (suppression theories). The former mechanism is called the permanent rivalry hypothesis. It is the central mechanism in the theory. It is shown that experimental results support it at the expense of suppression models as well as fusion models.

Both Julesz (1971) and Nelson (1975) have formulated a model of disparity-depth perception. Experimental results of Kaufman et al. (1973) falsify Nelson's model. The other model can escape this falsification by a more abstract and a more economical formulation. This reformulated model can be combined in a natural way with the permanent rivalry hypothesis. This combination is the starting-point of the theory.

In Chapter 2 a (two-dimensional) cyclopean space is defined. Two state variables, signal-type and disparity, are added to each point. The signal-type can have four values which indicate what is processed locally: a brightness signal from the left eye, from the right eye, a darkness signal from the left eye or from the right eye. The disparity indicates the locally perceived disparity. The actual state of the cyclopean system can be described by the cyclopean field-function which is a characteristic function of the state variables, space and time.

It is argued that the interaction mechanism of the cyclopean system

has a local character. It means that a stimulus is not processed as some combination of more elementary stimuli, although many experimental results seem to indicate the contrary. As an example experiments on spatial frequency perception are discussed at length. It is shown that their results, which seem to be contradictory, can be accounted for by other explanations. On the other hand phenomenological observations exist which support the hypothesis of the local character of the interactions. The state of a finite set of (N) points, homogeneously divided over the space, is supposed to be sufficient to describe the state of the system. Experimentally it has been shown that the system tries to align the state of these points, which are called cyclopean detectors. The part of the interaction which causes it, is called the co-operative interaction. The input of the system tries to align locally the signal-type of the detector with the signal-type of the stimuli. However, it is shown that some transitions between signal-type states are prohibited. It is supposed that permanent rivalry, in accordance with the allowed transitions, exists. (Only) signal-type transitions can cause changes in disparity due to a stimulus.

As a result of psychphysical noise subjects report the mean value of some quantity over a set of detectors in a small region of space. Therefore one can only formulate a stochastic theory about the interaction process. The state of the system is defined as (the Cartesian product of) the states of all detectors. The probability of finding the system in some state is defined. From this a stochastic process can be formulated which is supposed to be a Markov process. Psychophysically it means that only the actual percept - not the foregoing ones - and the stimulus determine the next percept. A dynamic equation - the master-equation - is formulated. by means of projection this equation can be converted into a dynamic equation of interactions between and within detectors. It is assumed that the interaction mechanism comprises three mutually independent mechanisms. One is the co-operative interaction, another is some autonomous disparity interaction and the third one is the rivalry, which is due to the input. The co-operative interaction is identified with the probability current density between detectors,

whereas the local transition-probability per unit time is partly identified with the retinal output, which expresses the change of the retinal input per unit time. This leads to an intricate set of 4N coupled differential equations. If N approaches infinity the summation over the detectors converts into an integration over the space, so that the cyclopean system is described by 4 differential-integral equations. This coupling expresses the permanent rivalry.

This set of equations, further indicated as the system's equation, contain four undetermined spatial functions. In Chapter 3 it is argued that the interaction-range of the function related to the co-operative interaction term is of the order of the Vernier-acuity and that of the function related to the autonomous disparity interaction is of the order of the stereoscopic acuity. The other two functions are related to the permanent rivalry interaction. One has to do with binocular interaction and its range is of the order of Panum's area. The other one governs brightness-darkness interaction. The range is very small. Probably it can be regarded as a Dirac-delta function.

Although the system's equation can hardly be solved, several conclusions can be drawn if one looks at stationary solutions. It is shown that all solutions tend to a stationary solution or a linear combination of stationary solutions of subsystems, if the input is time-independent. Often one can determine the characteristics of the stationary solution. It is shown that the stationary solutions in the case of monocular stimulation (one eye) and identical stimulation (both eyes) with the same stimulus are almost equal and that in both cases disparity-depth is not perceived. In the case of complementary (completely rivalrous) stimuli a unique stationary solution does not exist. It is shown that the system's equation can evoke perceptual hysteresis-phenomena. They are caused by the permanent rivalry interaction.

In Chapter 4 calculations on brightness-darkness perception are made. These non-trivial quantitative predictions can be made, although the theory is based on qualitative assumptions. It is argued that dark-

adaptation means that the output of the retinal system can be described as white noise. It leads to the perception of greyness: Eigengrau.

The theory predicts constancy of binocular brightness (up to first order) if the combination of the output-strengths of the retinal systems follow the centroid model. The latter is found to be the better model for describing binocular brightness mixing (De Weert & Levelt, 1974). It is shown that the criticism of Curtis and Rule (1978) on their results is false.

It is argued that subjects sometimes cannot make a comparison between the brightness of stimuli. It is supposed that then they compare lightness which is defined as the ratio of the brightness of an area and the weighted sum of the brightness of neighbouring areas. Thus experimental results of Jameson and Hurvich (1961) are predicted.

Only qualitative assertions can be made about binocular rivalry . In Chapter 5 it is argued that binocular rivalry is explained by the fact that in the case of complementary stimuli the system's equation has two, almost stable, solutions between which a transition is possible. Eye-movements induce several self-terminating processes which cause fluctuating transitions along stimulus contrast. However one process cannot halt and causes a total transition. Thus Levelt's (1968) model for alternation is supported.

In Chapter 6 the perception of disparity-depth is discussed. It is shown that in the case of stimulus disparity the stationary solution is a solution in which the perceived disparity equals the stimulus disparity if the latter lies within Panum's area. The results of the experiment mentioned earlier of Kaufman et al. (1973) are explained.

The theory explains several visual phenomena in a non-trivial way due to the principle of permanent rivalry. As it is argued in Chapter 0 this principle can be used possibly in other domains of perception as well. One can make second order developments of the co-operative interaction and the autonomous disparity interaction (Chapter 3). This amounts to

the formulation of a second order differential equation for both terms. One can analyse spatio-temporal and disparity-temporal relations with this model. Such an analysis (Buffart, 1978) agrees with experimental results. However, the development of special models by defining the spatial functions, mentioned in Chapter 3, is not an objective of the present study.

In dit proefschrift wordt een theorie ontwikkeld, die modellen over visuele verschijnselen als binoculaire rivaliteit, sensorische fusie en waarneming van diepte middels binoculaire dispariteit onderbouwt. Verscheidene onderzoekers hebben reeds verondersteld, dat deze verschijnselen te herleiden zijn tot één mechanisme: het cyclopische systeem.

De theorie beschrijft zowel structurele als quantitatieve aspecten. Quantitatieve aspecten worden mede bepaald door stimulus-transformaties in de retina. In hoofdstuk 0 wordt voorgesteld het retinale systeem opgebouwd te denken uit een algebraïsche niet lineaire transformatie van de stimulus-intensiteit L gevolgd door een spatio-temporele lineaire transformatie. De niet-lineaire transformatie is van belang voor de quantitatieve voorspellingen van de theorie over het cyclopisch systeem. Er zijn goede argumenten om te veronderstellen dat deze transformatie een quasi-logaritmische - $F_2(L)$ - is. Een van de argumenten is, dat deze transformatie een verklaring kan bieden voor het optreden van zowel Fechner's wet als Stevens' wet.

Bovendien levert een analyse van het ruimtelijk deel van de lineaire transformatie ook argumenten, die deze hypothese steunen. In Buffart (1978) is het spatiële deel opgevat als contrast-vervagend met een contrast-opscherpend mechanisme. De opscherping kan gezien worden als een correctie op de vervaging. De Fourier-getransformeerden van deze mechanismen zijn de eerste twee functies uit de verzameling orthogonale Laguerre-functies. De combinatie van $F_2(L)$ en dit spatiële mechanisme kan enige niet-lineaire verschijnselen bij de waarneming van rasters met een sinusvormige intensiteitsverdeling verklaren. Op dezelfde wijze kan

een niet-lineair effect in de temporele dempings-krommen van het visuele systeem begrepen worden.

De output van het retinale systeem hangt zodanig van de temporele veranderingen in de input af, dat bij een sterke ruimtelijke verandering van de stimulus-intensiteit van L' naar L de output evenredig is met $|F_2(L') - F_2(L)|$. Oogbewegingen kunnen langs een scherp ruimtelijk contrast een dergelijke output veroorzaken.

In hoofdstuk 1 worden enige modellen voor binoculaire onderdrukking, sensorische fusie en diepte-waarneming op grond van binoculaire dispariteit besproken. De kwalitatieve theorie van Sperling (1970) is opgebouwd uit een model voor fusie en een model voor onderdrukking. Experimentele resultaten van Levelt (1968) falsifiëren deze theorie. Na een bespreking van enige algemene bezwaren tegen fusie- en onderdrukkingstheoriën, wordt aangetoond dat de essentiële fout in fusie-modellen is, dat zij gebaseerd zijn op een mechanisme op micro-niveau, waarin het signaal afkomstig van het ene oog de signalen van het andere oog in een corresponderende omgeving onderdrukken. Er wordt voorgesteld om zulk een mechanisme te vervangen door een mechanisme, dat slechts locale onderdrukking toelaat en, in het geval van locale dominantie, de dominantie van signalen afkomstig uit hetzelfde oog bevordert. Dit mechanisme wordt de permanente-riv. iteit-hypothese genoemd. Het is het centrale mechanisme in de hier voorgestelde theorie. Experimenten blijken deze hypothese te steunen ten koste van fusie- en onderdrukkingstheoriën.

Julesz (1971) and Nelson (1975) formuleerden beiden een belangwekkende disparieit-detectie theorie voor diepte waarneming. Experimenten van Kaufman e.a. (1973) blijken Nelson's theorie te falsifiëren. Door het Julesz model abstracter en zuiniger te formuleren wordt het niet gefalsifieerd. In deze formulering kan het interactie-mechanisme van het model op natuurlijke wijze aansluiten bij de permanente-rivaliteit-hypothese. Deze combinantie is het uitgangspunt van de hier te ontwikkelen theorie.

In hoofdstuk 2 wordt een (twee dimensionale) cyclopische ruimte gedefiniëerd. Aan elk punt in deze ruimte worden twee

toestandsvariabelen - type signaal en disparıteıt - toegekend. Het type signaal kan vier waarden hebben. Deze geven aan wat lokaal wordt waargenomen: een helderheidstoename of een helderheidsafname afkomstig van het linker- of rechteroog. De dısparıteit geeft de locaal waargenomen dısparıteit weer. De toestand van het systeem wordt beschreven door de zogenaamde cyclopısche veld-funktıe, die een karakteristieke funktie van de toestandsvariabelen, de plaats en de tijd is.

Er wordt uitgegaan van de veronderstelling dat het cyclopısche ınteractıe-mechanısme een stimulus niet beschouwt als een combinatıe van meer elementaire stimuli, hoewel veel experimentele resultaten hierop schijnen te wijzen. Als voorbeeld worden experimenten over het waarnemen van spatiële frekwentıes uitvoerig besproken. De resultaten blijken elkaar niet alleen tegen te spreken, doch kunnen ook op geheel andere wijze verklaard worden. Bovendien bestaan er directe observaties die duiden op het bestaan van een mechanisme van locale interactie. Er wordt verondersteld dat de toestand van het systeem beschreven kan worden door de toestand van een eindig aantal (N) punten, die homogeen verdeeld zijn over de cylopısche ruimte. Zo'n punt wordt een cyclopısche detector genoemd. Experimenteel is reeds gebleken dat het systeem streeft naar identieke toestanden van de detectoren. Het mechanisme dat hiervoor zorgt wordt de coöperatıve interactie genoemd. De ınput van het systeem streeft er naar het systeem lokaal in een toestand te brengen, die overeenkomt met het type signaal van de input. Aangetoond wordt dat een overgang tussen sommige typen signalen verboden is. Verondersteld wordt dat permanente rivaliteit bestaat tussen toestanden met niet verboden overgangen, en dat slechts overgangen tussen typen signalen veranderingen in waargenomen dısparıteit ten gevolge van de stimulus mogelijk maken.

Het bestaan van psychophysische ruis leidt tot de conclusie dat proefpersonen rapporteren over de gemiddelde waarde van een of andere grootheid. Dit gemiddelde wordt bepaald door een verzameling detectoren in een klein deel van de ruimte. Derhalve kan slechts een stochastische theorie over het interactie-proces geformuleerd worden. De toestand van het systeem wordt gedefinieerd als (het Cartesisch product van) de

toestand van alle detectoren. De waarschijnlijkheid om het systeem in een bepaalde toestand aan te treffen wordt gedefinieerd. Op basis hiervan kan de interactie als een stochastisch proces gedefinieerd worden. Er wordt verondersteld dat het een Markov proces is. Dit betekent, dat slechts het aanwezige percept en de momentane stimulus het volgende percept bepalen. Op basis van deze veronderstelling kan een dynamische vergelijking - de zogenaamde master-equation - geformuleerd worden. Uit deze vergelijking kan een vergelijking voor de toestand van individuele detectoren worden afgeleid. Het interactie mechanisme bestaat uit drie onderling onafhankelijke mechanismen. Ten eerste de coöperative interactie, die beschreven wordt als een waarschijnlijkheidsstroomsterkte tussen detectoren. Ten tweede een autonoom mechanisme in de disparitеitsruimte. Ten derde een input-afhankelijk mechanisme dat de rivaliteit veroorzaakt. De input is de output van de beide retinale systemen en drukt de verandering van de stimuli per tijdseenheid uit. Dit leidt tot een stelsel van 4N gekoppelde differentiaal vergelijkingen. Als N naar oneindig gaat, gaat de sommatie over de detectoren over in een integraal over de cyclopische ruimte. Het cyclopisch systeem wordt dan beschreven middels 4 gekoppelde differentiaal-integraal vergelijkingen. De koppeling drukt de rivaliteit uit.

Dit stelsel wordt de systeemvergelijking genoemd. Het bevat vier nader te bepalen ruimtelijke funkties. In hoofdstuk 3 wordt betoogd dat de funktie die het bereik van de coöperatieve interactie bepaalt van de orde van de Vernier-scherpte is, en dat de funktie die het bereik van het autonome disparitеit-mechanisme vastlegt van de orde van de stereoscopische scherpte-grens is. De range van de derde funktie bepaalt de reikwijdte van de binoculaire rivaliteit en komt overeen met het Panum gebied. De vierde funktie beschrijft de reikwijdte van de licht-donker interactie. Deze is zeer klein en kan beschouwd worden als een Dirac-delta funktie.

Hoewel de systeem vergelijking moeilijk oplosbaar is, kunnen conclusies getrokken worden uit de stationaire oplossingen onder verschillende stimulus-condities. Alle oplossingen hebben als limiet (tijd naar

oneindig) een stationaire oplossing of een lineaire combinatie van stationaire oplossingen, als de input tijd-onafhankelijk is. De specifieke kenmerken van deze oplossingen kunnen dikwijls bepaald worden. Zo zijn de oplossingen in het geval van monoculaire stimulatie (één oog) en identieke stimulatie (beide ogen (te)gelijk) met de zelfde stimulus praktisch identiek en wordt er geen diepte tengevolge van disparitеit waargenomen. Als de stimuli complementair zijn (volledige rivaliteit) bestaat er geen unieke stationaire oplossing. De systeemvergelijking kan hysterese-verschijnselen in de waarneming verklaren als gevolg van de permanente rivaliteit.

Hoewel de theorie slechts gebaseerd is op kwalitatieve overwegingen, worden in hoofdstuk 4 onafhankelijke kwantitatieve voorspellingen over licht- en donker-waarneming gedaan. Er wordt verondersteld dat in het geval van donker-adaptatie de output van het retinale systeem witte ruis is. Dit leidt tot de waarneming van grijsheid, bekend onder de naam Eigengrau.

De theorie voorspelt dat binoculaire helderheidswaarneming in het geval van twee gelijkvormige monoculaire stimuli constant is (tot in eerste orde) als de output van beide retinale systemen aan het centroïd model voldoen. De Weert en Levelt (1974) beweerden reeds dat dit model tot nu toe het beste beschrijvingsmodel voor binoculaire helderheidscombinantie is. Aangetoond wordt dat recente kritiek (Curtis & Rule, 1978) op hun werk onjuist is.

Op basis van de veronderstelling dat, als proefpersonen slechts kunnen rapporteren over de relatieve helderheid van een stimulus-deel, dit oordeel gebaseerd is op de verhouding van de diverse helderheden, kunnen de resultaten van een experiment van Jameson and Hurvich (1961) voorspeld worden.

In hoofdstuk 5 wordt betoogd dat binoculaire rivaliteit een gevolg is van het feit dat de systeemvergelijking in het geval van complementaire stimuli twee bijna stabiele oplossingen heeft waartussen een overgang mogelijk is. Verscheidene zichzelf stoppende processen veroorzaken

locale en tijdelijke overgangen. Er is één onherroepelijk proces, dat de volledige overgang van de ene naar de andere oplossing veroorzaakt. Dit ondersteunt het model voor binoculaire alternering dat Levelt (1968) voorstelde.

In hoofdstuk 6 wordt aangetoond dat de stationaire oplossing in het geval van stimulus-dispariteit binnen Panum's gebied aanleiding geeft tot diepte-waarneming. De resultaten van het eerder vermelde experiment van Kaufman e.a. (1973) worden verklaard.

De theorie verklaart verschillende visuele verschijnselen op een niet triviale manier op basis van het beginsel van permanente rivaliteit. In hoofdstuk 8 wordt betoogd dat ook in andere perceptuele domeinen dit beginsel bruikbaar kan zijn. Tweede orde benaderingen van de coöperatieve interactie en de autonome disparitеit-interactie leiden tot tweede orde differentiaal vergelijkingen voor beide termen (hoofstuk 3). Op basis van zulke modellen kan men spatio-temporele en disparitеit-temporele relaties bestuderen. Conclusies uit deze modellen (Buffart, 1978) stemmen overeen met experimentele resultaten. Het is echter niet de opzet van dit proefschrift om specifieke modellen te ontwikkelen.

Curriculum vitae

H.F.J.M. Buffart werd geboren op 10 mei 1946 in Voorburg (ZH). In 1964 behaalde hij het Gymnasium-b diploma. Van 1964 tot en met 1970 studeerde hij theoretische natuurkunde en wiskunde aan de Rijksuniversiteit te Utrecht. In januari 1971 tradt hij in dienst van de Katholieke Universiteit te Nijmegen. Tot maart 1975 was hij wetenschappelijk medewerker bij de vakgroep Wijsgerige en Historische Pedagogiek. Sedertdien is hij werkzaam bij de vakgroep Psychologische Funktieleer. De eerste twee jaar deed hij onderzoek ten behoeve van het ZWO-projekt "Theoretische en experimentele studie naar de cyclopische perceptie" onder leiding van prof. dr. Th.G.G. Bezembinder en prof. dr. W.J.M. Levelt. Dit onderzoek vormt de basis van dit proefschrift. Van 1975 tot 1979 was hij adviseur bij het ZWO-projekt "Visuele informatie verwerkingscapaciteit bij zwakzinnigen". Van 1977 tot 1980 was hij, met dr. E.L.J. Leeuwenberg, projekleider van het ZWO-projekt "Generatie en selectie van patroon-codes". Sedert 1980 is hij, met dr. E.L.J. Leeuwenberg, leider van het ZWO-zwaartepuntprojekt "Structural information as a basis for understanding perception processes and for specifying human intelligence".

STELLINGEN

1. Op het eerste gezicht is een vierkant wèl een cirkel.
   Piaget, I., & Inhelder, B. La représentation de l'espace chez l'enfant. Paris: Presses universitaires de France, 1948.
   Buffart, H., Leeuwenberg, E., & Restle F. Analysis of ambiguity in visual pattern completion. Journal of Experimental Psychology: Human Perception and Performance, (in press).

2. Gestalt-regels zijn afdoende gefalsifiëerd.
   Buffart, H., Leeuwenberg, E., & Restle, F. Coding theory of visual pattern completion. Journal of Experimental Psychology: Human Perception and Performance, 1981, 7, 241-274.

3. Stimuli worden op minstens twee manieren tegelijk geïnterpreteerd.
   Buffart, H., & Leeuwenberg, E. Structural information theory. Leeuwenberg, E., & Buffart, H. An outline of experiments on structural information theory. Beiden in: H. Geissler, E. Leeuwenberg, S. Link, & V. Sarris (Eds.), Modern issues in perception. Berlin: 1981.

4. De falsificatie van suppressie-theorieën berust op het feit, dat daarin de waargenomen binoculaire rivaliteit geïnterpreteerd wordt als het aan deze rivaliteit ten grondslag liggende mechanisme.
   Dit proefschrift.

5. Het feit, dat binoculaire helderheid in eerste orde een gewogen som is van monoculaire helderheid is een gevolg van permanente rivaliteit.

Levelt W.J.M. On binocular rivalry. The Hague, Paris: Mouton, 1968.

Dit proefschrift.

6. Het aantrekkingsmechanisme in het door Julesz voorgestelde dipole model is een mechanisme voor de permanente rivaliteit.

Dit proefschrift.

7. Motorisch synchroniseren met min of meer regelmatige stimulus-intervallen berust op een extrapolatie van eerdere intervallen. De extrapolatie is een som van de gewogen gemiddelden van waargenomen intervallen en van de gewogen gemiddelden van waargenomen fouten. De wegings-coëfficiënten zijn dalende functies van de tijd.

Michon, J.A. A model of some temporal relations in human behaviour. Psychologische Forschung, 1968, 31, 287-298.

Buffart, H., & Vos, P. Performance in synchronization tasks with irregularly spaced temporal events: model and data. Internal report, 79 FU 09, Nijmegen University, 1979.

8. Het feit dat het produkt van drempelige stimulus-intensiteit (L) en oppervlakte (O) een U-vormige funktie is van O, wijst niet ondubbelzinnig op het bestaan van center-on surround-off cellen. De curve volgt uit de vergelijking die ontstaat door de output van het retinale systeem, zoals voorgesteld in Buffart (1978), constant te kiezen.

Buffart H. Brightness and contrast. In: E. Leeuwenberg, & H. Buffart (Eds.), Formal theories of visual perception. Chichester, New York: Wiley, 1978.

9. Een "Aha-Erlebnis" in de waarneming kan in termen van structurele informatie theorie begrepen worden als een verandering van de geprefereerde interpretatie van een object onder invloed van context, doordat de geprefereerde interpretatie van object en context tezamen verschilt van die van ieder afzonderlijk. De mechanismen voor abstraheren en waarnemen zijn niet onderscheidbaar *).

10. Twee gebeurtenissen kunnen door een waarnemer alleen dan met elkaar in causaal verband gebracht worden, als een van beide gebeurtenissen de geprefereerde interpretatie van de andere rechtstreeks verandert, maar niet omgekeerd. Derhalve gaat interpretatie van gebeurtenissen vooraf aan de toekenning van causale, en dus ook temporele orde tussen deze gebeurtenissen *).

*) De structurele informatie theorie leert, dat elk object beschreven kan worden door een code, die is opgebouwd uit vier basisfunkties welke slechts identiteit van elementen binnen een object vastleggen. Wat in een code niet middels identiteit kan worden beschreven, wordt de structurele informatie van een code genoemd en komt overeen met de onregelmatigheid in een object. In het algemeen zijn voor één object meerdere codes mogelijk. De code met de laagste hoeveelheid structurele informatie heet de minimum-code.

Anderzijds kan men met één code meerdere objecten beschrijven. Een code beschrijft een verzameling objecten. De theorie stelt dat er een één-éénduidige relatie tussen codes en interpretaties bestaat. Een interpretatie is een classificatie van een object als een element van de door de code gegenereerde verzameling. De interpretatie die gerelateerd is aan de minimum-code is de door een waarnemer geprefereerde interpretatie.

Een code dient altijd berekend te worden over het object en de context tezamen. Context is zowel andere tegelijk aangeboden objecten als reeds beschikbare kennis.

Dezè stellingen behoren bij het proefschrift:
A theory of cyclopean perception, H.F.J.M. Buffart
promotor: prof.dr. W.J.M. Levelt
Katholieke Universiteit Nijmegen, 21 januari 1982, 14.00 uur.

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